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THE ROTATED DIAGONAL FACTORS (RDF) APPROACH: A SUBSTITUTE FOR MANOVA WHEN ANALYZING MULTI-TASK AND MULTI-CRITERION DATA

by

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In the study examined, subjects (Ss) were required to perform a continuous two-dimensional tracking task whose criteria included RMS Error in both dimensions as well as the number of times the Ss manipulated the control stick in either dimension. Occasionally, one of two other discrete tasks (i.e., a tactical assessment task and a communications task) occurred. Performance on the discrete tasks included accuracy of responses as well as response time for both correct and incorrect responses. Over a period of time, this provided potential data about percent of correct, incorrect, and missed responses as well as median times for both correct and incorrect responses. Thus, several criteria were available for each of the three tasks. The tracking data analyzed in this study were limited to only those data which occurred during and immediately after the occurrence of either type of discrete task. This approach was taken because, in the absence of a second actual task, the tracking task was the only one which Ss had to actively perform, although Ss were responsible for monitoring if a new target had appeared which required their assessment.

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FOREWORD AND ACKNOWLEDGEMENTS

The development of an improved means for analyzing multi-task and multi-criterion data has been a concern for at least three decades. The Rotated Diagonal Factors (RDF) approach presented here is an outgrowth of the general dissatisfaction with the Multivariate Analysis of Variance (MANOVA) approach. The MANOVA approach is that it fails to address important and critical issues regarding correlations among tasks and criteria. Some sort of factor analytic approach is desirable to identify underlying dimensions that would be useful in understanding those relationships. Theoretically, there should be at least three types of independent factors that might be found. The first would be a between-tasks type that could account for significant criterion correlations between different tasks. The second would be a within-task type that could account for significant remaining relationships among different criteria for a single task not already accounted for by the first type of factor. The final would be a within-criterion type that would account for the remaining variance of a criterion not accounted for by the first two types. After identifying these types of independent factors, it would be desirable to establish if individual differences and experimental manipulations had significantly impacted all three types of factors.

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A method for mathematically rotating factors to the desired structure is being developed. This improvement will eliminate the necessity for manual graphical factor rotation, which, in turn, will make the RDF approach completely objective. Another enhancement which is under development is a knowledge-based program for the automatic interpretation of RDFs. This enhancement would permit the automatic recognition of RDFs that represent such concepts as "time-sharing," "attention shifting among tasks," "response speed," "response accuracy," "speed-accuracy trade-offs," etc. While these developments have not, as yet, been completed, the basic concepts and procedures for the RDF approach have now been developed and tested to a point where they can be used by other researchers who are faced with the analysis of multi-task and multi-criterion data.

1. INTRODUCTION

Many human factors studies involve performance of simulated tasks under varying experimental conditions. Major questions raised in these studies almost always concern how much the experimental conditions enhance or degrade task performance and whether performance differences are statistically significant. Analysis of Variance (ANOVA), the method traditionally used to determine this impact, partitions total task performance into independent components. Historically, the major variance components have included subject effects, (i.e., variance attributable to individual or group differences), experimental effects (i.e., variance attributable to various experimental manipulations), and residual effects (i.e., unexplained variance usually attributed to error). The ANOVA technique also computes model coefficients for various levels of subjects and experimental manipulations and tests their significance to determine the probability those differences happened by chance alone.

A new analytic technique to be discussed and demonstrated in this paper is one that extends the concept of partitioning variance into yet further independent components when the study involves multiple tasks and multiple performance criteria. It combines traditional multivariate ANOVA techniques and factor analytic techniques. The technique is referred to as the Rotated Diagonal Factors (RDF) approach. It is expected to be particularly useful for situations when performance on a single task is measured on more than one criterion, when performance on more than one task is measured, or a combination of these situations.

1.1 Need to Consider Multiple Task Situations

Experimenters often assume that effects in laboratory studies of a single task will also be obtained when that task is accomplished in the real world. More often than not, however, an isolated task studied under laboratory conditions is only one of several ongoing tasks for which an operator may be responsible under real world situations. While task demands influence what needs to be done, operators ultimately determine how much attention they dedicate to each

ongoing task responsibility. Indeed, ability to rapidly adapt behavior to momentary task-related attentional demands is one of the strengths of human operators. This ability to time-share attention among several tasks suggests that conclusions based on situations in which only a single task was performed may not be applicable to real world, multi-task conditions. If true, task performance should be studied in the context of the totality of tasks to be performed. It seems obvious that specific experimental manipulations (e.g., different display/control designs, crewstation layouts, seating, restraints, etc.) may enhance performance on one task, but, at the same time, lead to degraded performance on another. Other situational conditions (e.g., levels of noise, illumination, vibration, G-forces, etc.) may differentially enhance or degrade performance across several tasks. The likelihood of such outcomes strongly suggests that instead of merely considering the variance in performance of each separate task in isolation, investigators should analyze their data for potential between-tasks effects. Between-tasks effects can be defined as "those effects which cause performance on one or more criteria of one task to covary (positively or negatively) with performance on one or more criteria for other tasks."

1.2 Need to Consider Multiple Criteria for a Single Task

When considering performance on a single task, conclusions drawn about either individual differences or experimental manipulations may be contingent on how task performance is measured. In a target recognition task, for instance, analyzing only "percent of correct recognitions" may lead to quite different conclusions than the analysis of "decision response time." Indeed, even "response times for correct decisions" are often different from "response times for incorrect decisions." Additionally, it has long been accepted that humans are capable of time-accuracy trade-offs, and, unless the experimenter includes multiple criteria for evaluating task performance, the impact of both individual differences and experimental manipulations during a laboratory study may not be appreciated fully. Thus, even when studying only a single task, investigators may need to analyze performance variability among several criteria for that task to determine the presence of significant within-task effects. Within-task effects can be defined as "those effects which cause performance on one criterion of a task to covary (positively or negatively) with another criterion measure of that same task."

Finally, in addition to potential between-tasks effects and within-tasks effects, it is also possible to assume that individual differences or some experimental manipulation may effect only a single criterion of a single task. This type of effect is defined here as a within-criterion effect.

Thus, theoretically, in a multi-task, multi-criterion study, the total variance in performance on each criterion should be able to be divided into three types of independent variance components: (a) between-tasks, (b) within-task (but between criteria), and (c) within-criterion. The Rotated Diagonal Factors approach, to be discussed later, accomplishes this objective.

1.3 Individual Differences in Task Performance

A "proficient" task performer is usually defined as one who can respond fairly rapidly and makes few or no errors. Conversely, one who responds slowly and makes many errors is almost always considered to be a "non-proficient" task performer. If subjects (Ss) in a study were only composed of these two styles of people, then performance measures of response time and response accuracy would always be negatively related across subjects (i.e., those with short response times would have high accuracies and those with long response times would have low accuracies). However, there may also be those who might be described as "careful" individuals who tend to make few errors but have relatively long response times or those who might be described as "decisive" and who respond extremely fast but make somewhat more errors. If Ss in a study were only composed of these two styles of people, then performance measures of response time and response accuracy would always be positively related across subjects (i.e., those with high response times would have high accuracies and those with low response times would have low accuracies). Of course, in most studies, all four styles of people may be present and the direction of the relationship between response time and response accuracy is difficult to predict.

Task difficulty can also affect the relationship between response time and response accuracy. For example, more difficult decisions may take more time and have a higher likelihood of error. The greater the range of item difficulty on a particular task, the more likely it is for response time and response accuracy to have a negative relationship. Also, in a multi-task situation, difficulty level for one task may influence the available time Ss have to attend to another task.

Amount of training can also effect the relationship between response time and response accuracy. With more and more training and experience, people tend to become more proficient, but individual differences between "fully trained" persons, caused by inherent capabilities between those persons, may still be present. Even in high demand, multi-task situations, humans can learn to recognize the urgency and criticality of decision tasks and appropriately trade-off response time for response-accuracy. They will usually take somewhat longer times to make more difficult and critical decisions, but they can also usually recognize when decisions must be made even though they might have preferred more time to consider their responses. Thus, the level of task demands may well affect the direction and magnitude of the relationship between response time and response accuracy.

Any specific task on which performance can be measured undoubtedly requires several different types of processes (e.g., perception of the stimuli, cognitive processing of those stimuli, production of a response, etc.). When two different tasks require similar processes, one would expect those who are more gifted in those specific capabilities to do somewhat better on both tasks than those who are less gifted. Thus, individual differences in similarly required task processes should lead to positive covariance between criteria for two tasks. Enhanced proficiency in performing any specific task could also arise through training and practice on that task. Thus, even if two tasks did not share any of the same required capabilities, more training and practice on both tasks for some individuals than others could lead to positive criterion covariance between them. Finally, it is reasonable to assume that the level of performance produced by an individual on any task could also be effected by that individual's general level of motivation, fatigue, or environmental factors such as levels of distraction. Thus, certain general

effects, not directly related to any of the tasks, could also result in positive covariance among criteria for two tasks.

1.4 The Challenge of Multiple Task and Multiple Criterion Studies

From the foregoing discussions, it follows that between-tasks effects may be found which are attributable to individual differences (either those that result from: (a) inherent capabilities, (b) differential training and practice, or (c) general factors such as motivation, fatigue, or distractions). Each of these could cause differences in the level of task proficiency of subjects which result in positive covariance among criteria for different tasks. Thus, while many between-tasks factors may be primarily caused by individual differences, during a repeated measures study, specific experimental manipulations (e.g., number of sessions, length of sessions, order of conditions within sessions, and periods within sessions) could all be expected to impact the extent of practice and conditions of fatigue and, therefore, the subject's task proficiencies.

Within-task effects could also result from individual differences in capabilities not used in other tasks and could also represent individual differences in trading-off speed and accuracy. A particular experimental condition might, conceivably, affect how one particular task gets accomplished, but have little effect on other tasks.

If human factors related findings are expected to generalize to the real world, then researchers must, for many situations, investigate performance on multiple ongoing tasks. If they are to fully understand how behavior is being affected by both individual differences and various experimental manipulations, then they must also include multiple criteria for all of those tasks. While MANOVA offers a method for analyzing multi-task and multi-criterion effects, it is not sufficient to isolate and identify the between-tasks, within-task, and within-criterion effects discussed earlier. The RDF approach was developed for this specific purpose.

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2. MULTIVARIATE TECHNIQUES

In this section, similarities and differences among six multivariate techniques used in experimental data analysis will be discussed. These multivariate techniques and the subsections in which they will be discussed are:

2.1 Analysis of Variance (ANOVA);
2.2 Multiple Correlation (MC) and Multiple Regression (MR);
2.3 Diagonal Factor Analysis (DFA);
2.4 Multivariate Analysis of Variance (MANOVA);
2.5 Factor Analysis (FA) and Factor Rotation (FR) methods; and
2.6 Canonical Analysis (CA)

The reason for describing these techniques in some detail is that the RDF approach, which will be discussed in subsection 2.7, is an extension of these techniques and of the logic and principles upon which they are based.

2.1 Analysis of Variance (ANOVA)

Textbooks on ANOVA spend much time showing experimental designs which cause the main effects and interaction terms to be independent of one another, how to determine the degrees of freedom associated with main effects and interaction terms, how to calculate the various independent portions of variance associated with those factors, and how to test their significance. The lack of space here prohibits any detailed discussion of the ANOVA approach, but the major principles underlying ANOVA will be discussed. For that discussion, an ANOVA study having two main effects (**A** and **B**) and an interaction term (**AB**) will be used as an example. We can think of a data matrix in which we have rows representing the different levels of **A**, columns representing the different levels of **B**, and the cells of the matrix representing the levels of **AB** interactions between the two main effects.

main effects		**B levels**					
		b_1	b_2	...	b_j	...	b_B
	a_1	ab_{11}	ab_{12}	...	ab_{1j}	...	ab_{1B}
	a_2	ab_{21}	ab_{22}	...	ab_{2j}	...	ab_{2B}
A	...	...	...	...	...	...	...
levels	a_i	ab_{i1}	ab_{i2}	...	ab_{ij}	...	ab_{iB}
	...	...	...	...	...	...	...
	a_A	ab_{A1}	ab_{A2}	...	ab_{Aj}	...	ab_{AB}

One ANOVA principle is that the two main effects can be forced to be independent of one another by requiring, for each level of **B**, an equal (or, at least, proportional) number of data cases to be collected for a given level of main effect **A**.

A second principle used by ANOVA is that the actual (obtained) score for any data case is simply the sum of an overall effect (μ, i.e., the Greek letter "mu") plus the sum of the actual effects of each specific row, column, and cell and an error term for that data case. This concept is usually expressed as the "structural equation" for a particular design. For the case of two main effects, the structural equation for the obtained score of case **k** in row **i**, column **j** would be:

$$Y_{ijk} \quad = \quad \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + e_{ijk} .$$

This equation is related to a general statistical principle that the mean or average of any set of numbers is the best prediction (to minimize the sum of the errors squared) one can make for that set of numbers. ANOVA uses this principle by assuming that the true value of the overall effect or any level of main effect or interaction term is the weighted mean of the matrix, or appropriate row, column, or cell of the matrix (where the respective means are weighted by the number of cases in the respective matrix, row, column, or cell). If one calculates the overall weighted mean (μ) of all cases in the data matrix, it would be the best prediction one could make if all one knew was that the data case came from that sample. In ANOVA, μ is assumed to represent the true overall effect. Its value could then be subtracted from each data case score. The weighted residual score means of rows and columns are assumed by ANOVA to represent the true effects for the various levels of the two main effects (α_i and β_j). These score means, for each row and column, are then subtracted from each data case residual score in that row and column and the

means of each matrix cell are then calculated. These residual cell means are assumed by ANOVA to represent the true effects of the interaction ($\alpha\beta_{ij}$) between the various levels of the two main effects. These terms are shown below.

main effects				**B levels**			
	μ	β_1	β_2	...	β_j	...	β_B
	α_1	$\alpha\beta_{11}$	$\alpha\beta_{12}$	...	$\alpha\beta_{1j}$	...	$\alpha\beta_{1B}$
	α_2	$\alpha\beta_{21}$	$\alpha\beta_{22}$	...	$\alpha\beta_{2j}$	...	$\alpha\beta_{2B}$
A	...	...	...	...	...	...	...
levels	α_i	$\alpha\beta_{i_1}$	$\alpha\beta_{i_2}$	...	$\alpha\beta_{ij}$	...	$\alpha\beta_{i_B}$
	...	...	...	...	...	...	...
	α_A	$\alpha\beta_{A1}$	$\alpha\beta_{A2}$	...	$\alpha\beta_{Aj}$	...	$\alpha\beta_{AB}$

Finally, the appropriate cell means are subtracted from each residual data case score and the result is assumed by ANOVA to be the actual error value ($\mathbf{e}_{ijk}$) for each data case.

A third principle used by ANOVA is a general statistical proof that, in the long run (i.e., if an infinite number of random samples were drawn), the true variance of the means (σ^2_M) of random samples of **N** cases all drawn from the same population will be the true variance of that population (σ^2) divided by the number of cases (**N**) in each sample (i.e., $\sigma^2_M = \sigma^2 / \mathbf{N}$).

A fourth principle used by ANOVA is also a general statistical proof that, if an infinite number of samples of **N** cases are drawn at random, the expected value of the variance ($\mathbf{s}^2$), if multiplied by the quantity **N/(N-1)**, will be equal to the true population variance (σ^2). That is,

$$\text{Average } (\mathbf{s}^2 \times \mathbf{N/(N-1)}) = \sigma^2.$$

ANOVA combines these two principles by initially assuming that the levels of each main effect and interaction have no real effect (this is generally referred to as the null hypothesis). The earlier discussed procedures for finding the various means permits one to analyze the total variance (hence the name "Analysis of Variance") into independent, additive components (i.e., variance attributable to the first main effect means, variance attributable to the second main

effect means, variance attributable to the means of the interaction of the main effect, and variance attributable to "residual" error). If the null hypothesis is true, then the variance of each set of means can be used to get an unbiased estimate of the true population variance. That is,

$$s^2_Y = s^2_\alpha + s^2_\beta + s^2_{\alpha\beta} + s^2_{error}.$$

The traditional ANOVA equations for computing what are referred to as the sums of squares are those that compute functions of the variance of the appropriate sets of means (i.e., A, B, AB, and error). The equations for computing what are referred to as mean squares are those which derive estimates of the population variance based on the number of the means being considered. The **F**-ratios computed by ANOVA are simply the ratios of appropriate mean squares (i.e., population variance estimates) and are referred to as such because the **F** statistic reports the likelihood of such ratios when the null hypothesis is true. That is, **F** tables show the probability of the ratio of two independent estimates of variances based on known number of cases drawn from the same normally distributed population, being as large as the tabled values.

2.2 Multiple Correlation (MC) and Multiple Regression (MR)

Let us suppose that the correlation between two variables (**Y** and **X**) is calculated by a variation of the Pearson Product-Moment Correlation equation (i.e., $\mathbf{r}_{YX} = \mathbf{z}_Y\mathbf{z}_X/\mathbf{N}$) where $\mathbf{z}_Y$ and $\mathbf{z}_X$ are the standardized scores for variables **Y** and **X** respectively, and **N** is the number of cases in the sample. It can be shown that, when this is done, the square of that correlation coefficient (i.e., $\mathbf{r}^2_{YX}$) yields the proportion of variance of variable **Y** (a criterion variable) that can be explained by variable **X** (a predictor variable). It can also be shown that the "best" linear prediction (where "best" is defined as that which minimizes the sum (or average) of the squared errors of the predictions) of $\mathbf{z}_Y$ (i.e., $\mathbf{z}'_Y$) will be $\mathbf{r}_{YX}\ ^x\ \mathbf{z}_X$.

Multiple correlation is simply an extension of simple correlation for cases in which scores for a criterion variable, **Y**, are predicted using more than one predictor variable (e.g., $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$, etc.). That is, letting β represent a weight for multiplying standard scores, then the equation for **Y**'s predicted standard score ($\mathbf{z}'_Y$) for case **n** will be:

$$z'_{Yn} \quad = \beta_1 \times z_{X1n} + \beta_2 \times z_{X2n} + \ldots + \beta_m \times z_{Xmn}\,.$$

Thus, the purpose of multiple correlation is to derive weights for multiplying the standard scores of the predictor variables so as to obtain a prediction of the standard scores of the criterion variable, **Y**, which minimizes the sum (or average) of the squared errors of prediction. Because the means and standard deviations of the criterion and all of the predictor variables and the criterion are known, it is relatively simple, once the "best" standard score weights are derived, to then calculate a raw score prediction equation for predicting the raw score of variable **Y** for case **n**. That is, letting **Y'** be the predicted raw-score of variable **Y** and substituting $(\mathbf{X}_{in} - \mathbf{M}_{Xi}) / \mathbf{s}_{Xi}$ for each $\mathbf{z}_{Xin}$ and $(\mathbf{Y}'_n - \mathbf{M}_Y) / \mathbf{s}_Y$ for $\mathbf{z}'_{Yn}$ in the above equation:

$$(Y'_n - M_Y) / s_Y = \beta_1(X_{1n}-M_{X1})/s_{X1}+\beta_2(X_{2n}-M_{X2})/s_{X2}+ \ldots +\beta_m(X_{mn}-M_{Xm})/s_{Xm}\,.$$

Multiplying both sides by $\mathbf{s}_Y$ and adding $\mathbf{M}_Y$ to both sides,

$$Y'_n = M_Y+\beta_1 s_Y/s_{X1}(X_{1n}-M_{X1})+\beta_2 s_Y/s_{X2}(X_{2n}-M_{X1})+\ldots+\beta_m s_Y/s_{Xm}(X_{mn}-M_{X1})\,.$$

Letting $\mathbf{B}_i = \beta_i \mathbf{s}_Y/\mathbf{s}_{X_i}$, and collecting constants together,

$$\begin{aligned} Y'_n \quad &= M_Y + B_1(X_{1n}-X_1) + B_2(X_{2n}-X_2) + \ldots + B_m(X_{mn}-X_m)\,, \\ &= M_Y - (B_1M_{X1}+B_2M_{X2}+\ldots+B_mM_{Xm}) + B_1X_{1n}+B_2X_{2n}+\ldots+B_mX_{mn}\,. \end{aligned}$$

And letting $A = M_Y - (B_1M_{X1}+ B_2M_{X2} + \ldots + B_mM_{Xm})$, then

$$Y'_n \quad = A + (B_1 X_{1n} + B_2 X_{2n} + \ldots + B_m X_{mn})\,.$$

The only difference between multiple correlation (MC) and multiple regression (MR) is that, in doing MC, raw data for both the predictors and the criterion variable are treated as if they had first been converted to standard scores. Because both techniques are mathematically identical, the derived raw score or standardized weights and results of testing their significance will be identical regardless of whether one uses MC or MR.

The correlation between the actual and predicted criterion scores (i.e., $\mathbf{r}_{Y\underline{Y}'}$) would yield what is referred to as the multiple correlation coefficient. To indicate that **Y'** is composed of weighted portions of **m** predictor variables, rather than using $\mathbf{r}_{YY'}$, the multiple correlation coefficient is symbolized as $\mathbf{R}_{Y\ X1,X2,\ldots,Xm}$, or $\mathbf{R}_{Y\ 1,2,\ldots,m}$, or more simply, $\mathbf{R}_{Y\ 1\ldots m}$, or even more simply as **R**. Its square, $\mathbf{R}^2$, is equal to the amount of **Y** variance that can be accounted for by the prediction equation. The final multiple correlation coefficient can be tested for significance using an **F**-test where:

$$F_{(df = m,\ N-m-1)} = (R^2 / m) / ((1 - R^2) / (N-m-1)) .$$

It can be seen that the **F** value is the ratio of the total explained variance (i.e., $\mathbf{R}^2$) divided by the number of predictor variables (i.e., **m**) and the unexplained variance (i.e., $\mathbf{1\text{-}R}^2$) divided by **N-1** minus the number of predictor variables. But each new predictor variable selected represents a potentially new independent source of explained variance and can be tested separately. The significance of the **k**th predictor is also determined by an **F**-test. The numerator of this test is the difference in variance explained by the total **k** variables and that which had previously been explained by **k-1** variables. The denominator for this test is the unexplained variance based on all **k** variables divided by its appropriate degrees of freedom. That is,

$$F_{(df:\ 1,N-k-1)} = ((R^2_{Y\ 1,\ldots,k} - R^2_{Y\ 1,\ldots,k-1}) / 1) / ((1 - R^2_{Y\ 1,\ldots,k})/(N-k-1)) ,$$

$$= (R^2_{Y\ 1,\ldots,k} - R^2_{Y\ 1,\ldots,k-1}) / ((1 - R^2_{Y\ 1,\ldots,k})/(N-k-1)) .$$

A basic problem recognized in both MC or MR is determining how many predictor variables to include in the prediction equation. There are three different general solutions to this problem. The first solution's approach is referred to as the method of accretion (or "test selection"). It first selects the best single predictor (i.e., the one that will account for the most criterion variance), removes the effect of that variable from the matrix, and then looks for the next predictor that will explain the most residual variance of the criterion, and so on. Each succeeding **F** value is computed and can be tested by the above equation. If, at any time, in this sequential approach the amount of newly explained variance is not deemed to be significant, then the method stops at that point without selecting the non-significant variable.

The second solution's approach is referred to as the method of deletion. It selects all predictors and then, again using a variation of the above equation, tests the significance of each predictor variable. If one or more predictor variables is non-significant, the least significant variable is eliminated and the entire process is then repeated without that variable. This process continues until all predictor variables being used show a significant contribution. It is of passing interest that the first two methods usually arrive at the same conclusions as to which variables should be selected for the prediction equation.

The third solution's approach can be referred to as the select all predictors method. It selects all predictors, regardless of whether they are significant or not, and reports their levels of significance. This third method corresponds closely to traditional ANOVA in that ANOVA also uses all possible predictors (i.e., all levels of main effects and interaction terms in its structural equation). One impact of this is that the structural equation values derived by ANOVA may be over-fitting error and, thus, may not be as effective in predicting the criterion in future samples of similar data.

Many people think that MC and ANOVA are different ways to analyze data. Such is not the case. While ANOVA equations for obtaining sums of squares and mean squares are interesting from an historical standpoint, MC can be used to accomplish the same purpose as the traditional ANOVA approach. To accomplish ANOVA using MC requires the creation of

dichotomous variables for each degree of freedom in the ANOVA model. Data for any dichotomous variable is coded as "0" if the trait in question is not present and "1" if the trait is present. For example, if three levels of factor A are used in a given study, any two of the three levels could be used as the basis for coding the needed two dichotomous variables. For example, one variable to represent "Level A_1" and one variable to represent "Level A_2" could be created. Suppose, in this same study, there is a second factor, B, which also has three levels. Two dichotomous variables (B_1 and B_2) to represent this factor could also be created. Finally, four dichotomous variables to represent the interaction of these variables (i.e., A_1B_1, A_1B_2, A_2B_1, and A_2B_2) could be created. The coded data for the eight created dichotomous variables for all possible combinations of the different levels of condition and order are shown below. It can be seen that each combination gets assigned a different set of "0" and "1" values across the eight dichotomous variables, even though specific dichotomous variables do not exist for A_3, B_3, A_1B_3, A_2B_3, or A_3B_1, A_3B_2, or A_3B_3.

		created dichotomous (predictor) variables							
A Level	**B Level**	**A_1**	**A_2**	**B_1**	**B_2**	**A_1B_1**	**A_1B_2**	**A_2B_1**	**A_2B_2**
1	1	1	0	1	0	1	0	0	0
1	2	1	0	0	1	0	1	0	0
1	3	1	0	0	0	0	0	0	0
2	1	0	1	1	0	0	0	1	0
2	2	0	1	0	1	0	0	0	1
2	3	0	1	0	0	0	0	0	0
3	1	0	0	1	0	0	0	0	0
3	2	0	0	0	1	0	0	0	0
3	3	0	0	0	0	0	0	0	0

It is worth noting that no information is lost by not having dichotomous variables for "Level A_3," "Level B_3," or any of the interactions of those levels. The same rationale which permits elimination of those levels is identical to the rationale in ANOVA for "Effect A" having only two degrees of freedom (i.e., $\mathbf{df}_A$ = **(a-1)**), "Effect B" having only two degrees of freedom (i.e., $\mathbf{df}_B$ = **(b-1)**), and the interaction of those terms having only four degrees of freedom (i.e., $\mathbf{df}_{AB}$ = **(a-1)(b-1)**).

To accomplish MC, correlation coefficients among these predictor variables (**X**s) and the criterion variable (**Y**) are first computed. Multiple correlation is then accomplished using the predictors to explain the criterion variable. In performing the MC approach, the $\mathbf{R}^2$ will keep increasing as each predictor variable is utilized to improve the prediction. When using MC to accomplish ANOVA, the presentation of "the source table" results from the MC analysis would be equivalent to those shown below.

Source	**d.f.**	**Sums of Squares (SS)**	**Mean Squares (MS)**	**F-ratios**
Effect A (A)	2	$R^2_{Y\,1,2}$	$SS_A / 2$	MS_A/MS_E
Effect B (B)	2	$R^2_{Y\,1,2,3,4} - R^2_{Y\,1,2}$	$SS_B / 2$	MS_B/MS_E
Interaction AB	4	$R^2_{Y\,1,\ldots,8} - R^2_{Y\,1,2,3,4}$	$SS_{AB} / 4$	MS_{AB}/MS_E
Error (E)	N-8-1	$1 - R^2_{Y\,1,\ldots,8}$	SS_E / (N-8-1)	
Total	N-1	1.0		

Note: alternative F-ratios for the two main effects would be MS_A/MS_{AB} and MS_B/MS_{AB}.

With the MC approach which reports variance in terms of **z**-scores rather than raw-scores, the sums of squares (SSs) column can be seen, perhaps even more clearly than with traditional ANOVA source tables, to be partitioning the total variance of the criterion into the independent parts which were caused by the ANOVA design. Here it can be easily seen that the total sums of squares adds up to 1.0 which shows that the total SSs for both main effects, the interaction effects, and the error term accounts for all of the criterion variance. These SSs will be identical to those obtained by ANOVA if the criterion variable scores had first been converted to standard scores. It should also be noted that the degrees of freedom (**df**s) are equal to the number of dichotomous variables used to represent the various effects.

The approach of using dichotomous variables to represent the levels of main effects and interaction terms has been covered here in detail because this approach will also be used later in the Rotated Diagonal Factors (RDF) approach.

2.3 Diagonal Factor Analysis (DFA)

A matrix of correlation coefficients shows the relationships among the variables represented in that matrix. Each column of the matrix represents a vector having a variance of one and each row value shows the relationship of one variable with the column vector. These vectors, however, are not necessarily independent of one another. Diagonal Factor Analysis (DFA) is a method to obtain a set of independent factors which contains all of the same variance, and, at the same time, to obtain the relationship of each variable to those independent factors. The DFA method for doing this is quite simple. It sequentially derives successive independent factors and removes its effect from the matrix. Each independent diagonal factor **k**, for example, is found by dividing the remaining entry in column **k** by the square root of value in variable **k**'s main diagonal cell. These values become the diagonal factor loadings (**d**'s) for the first factor. That is, the diagonal factor loading of variable **i** on factor **k** will be:

$$d_{ik} \quad = r_{ik} / (r_{kk})^{.5} \ .$$

The effect of diagonal factor **k** is then removed from the $\mathbf{m}^{\mathrm{x}}\mathbf{m}$ correlation matrix using the equation for computing residual correlations before the next diagonal factor is found. The equation for finding the residual correlation by removing the effect of an independent factor, which is applicable for all rows and columns, is:

$$r_{ij\,residual} \quad = r_{ij} - d_{ik} {}^{\mathrm{x}} d_{jk} \ .$$

The process of finding diagonal factors and removing their effects is repeated until all variables have been selected and **m** diagonal factors have been found. It should be mentioned here that the first diagonal factor is identical to the first predictor variable. The second diagonal factor is equal to the second predictor when all the relationships with the first predictor has been removed (symbolized $\mathbf{X}_{2.1}$). The third diagonal factor is equivalent to the third predictor with all the relationships attributable to the first two predictors removed (symbolized $\mathbf{X}_{3.1,2}$), and so forth. Thus, symbolizing the **i**th diagonal factors as $\mathbf{D}_i$, then $\mathbf{D}_i = \mathbf{X}_{i.1,\ldots,i-1}$.

The above steps used in accomplishing DFA is actually one method for accomplishing MC. For example, if one diagonally factors the predictor variables in a matrix containing **m** predictors and a criterion variable, **Y**, it can be shown that

$$R^2_{Y\,1\ldots m} \quad = \quad d^2_{YX1} + d^2_{YX2.1} + \ldots + d^2_{YXk.1,\ldots,k-1} + \ldots + d^2_{YXm.1,\ldots,m-1}\text{, or}$$

$$\underset{i=1 \text{ to } m}{R^2_{Y\,1\ldots m}} \quad = \quad (d^2_{YXi.1,\ldots,i-1}).$$

where:

$R^2_{Y\,1\ldots m}$ = the squared multiple correlation for **Y** using **m** predictor variables,
$d^2_{YXi.1,\ldots,i-1}$ = the squared correlation of **Y** with $\mathbf{X}_i$ with all relationships to $\mathbf{X}_i$ through $\mathbf{X}_{i-1}$ removed.

Thus, these diagonal factors show the incremental increase in **Y** variance explained as each new predictor variable was selected, and each $\mathbf{d}^2$ is independent from all others. Thus, each $\mathbf{d}^2$ can also be seen to be an independent component of explained variance since $\mathbf{R}^2$ is the total variance explained.

2.3.1 Matrix Algebra to Describe the Operations for DFA and MC

Matrix algebra permits one to indicate various kinds of matrices. A matrix is a rectangular set of values having rows and columns. A square matrix is one where the number of rows equals the number of columns. A matrix is symbolized using brackets surrounding a matrix identifier (e.g., [**R**] indicates a correlation matrix, [**F**] indicates a factor matrix). Some special types of matrices are always symbolized in the same way. For example, [$\mathbf{0}_{mn}$] is referred to as a zero matrix and indicates an $\mathbf{m}^x\mathbf{n}$ matrix where each cell value equals zero; [$\mathbf{I}_{mm}$] is called an identity matrix and always indicates an $\mathbf{m}^x\mathbf{m}$ square matrix with "1" in each of the main diagonal (upper left to lower right) cells and "0" in all other cells. Matrix algebra can also be used to indicate various types of operations on those matrices. Matrices, if they have the proper number of rows and columns, can be added (i.e., [A]+[B]), subtracted (i.e., [A]-[B]), or multiplied (i.e., $[A]^x[B]$). One matrix cannot be divided by another, but an operation called matrix inversion, which is discussed in the next subsection, can be done.

The actual procedure for accomplishing multiple correlation (MC) (i.e., when there is only one criterion variable), or multi-criterion multiple correlation (MMC) (i.e., when there is more than one criterion variable), involves the same matrix operations. These operations can be explained using matrix algebra terminology. The overall matrix of correlations among a set of **m** predictor variables (**X** variables) and **n** criterion variables (**Y** variables) can be symbolized in matrix algebra as [**R**]. However, [**R**] can also be thought of as being partitioned into four submatrices, such that:

$$[\ \mathbf{R}\ ] \quad = \quad \begin{matrix} [\ \mathbf{R}_{XX}\ ] & [\ \mathbf{R}_{XY}\ ] \\ [\ \mathbf{R}_{YX}\ ] & [\ \mathbf{R}_{YY}\ ] \end{matrix} \quad ,$$

where:

[$\mathbf{R}_{XX}$] = the $\mathbf{m}^{x}\mathbf{m}$ correlations among the **m** predictor variables,
[$\mathbf{R}_{XY}$] = the $\mathbf{m}^{x}\mathbf{n}$ correlations of the **m** predictors with the **n** criteria,
[$\mathbf{R}_{YX}$] = the $\mathbf{n}^{x}\mathbf{m}$ correlations of the **n** criteria with the **m** predictors, and
[$\mathbf{R}_{XX}$] = the $\mathbf{n}^{x}\mathbf{n}$ correlations among the **n** criterion variables.

In matrix algebra, [$\mathbf{R}_{YX}$] is also known as the transpose of [$\mathbf{R}_{XY}$] and can also be symbolized as [$\mathbf{R}'_{XY}$]. To transpose a matrix, the left to right entries in each row of the original matrix become the top to bottom entries in the columns of the transposed matrix. In traditional multiple correlation, there is only one criterion variable (i.e., **n** = **1**). When this is true, as in ANOVA and MC, [$\mathbf{R}_{XY}$] is a single column (i.e., a vector) of correlations of the predictor variables with the criterion variable, and [$\mathbf{R}_{YY}$] would be a one-celled matrix containing the value 1.00 (i.e., the correlation of the criterion variable with itself). In the following derivations, multiple criteria (i.e., $\mathbf{n}\ ^{2}\ \mathbf{1}$) are permitted and this approach is referred to as Multi-Criterion Multiple Correlation (MMC).

2.3.2 Matrix Inversion

If the entries in each row and column of the criterion variables are reflected (i.e., all of their signs are changed), the top half of the matrix [**R**] becomes the set of simultaneous equations that must be solved to find the best weights for multiplying the **X** variables to predict the **Y** variables (i.e., the weights to multiply the predictor **z**-scores to best predict the **z**-scores of the criterion variable(s) whose relationships with the predictors are shown in [$\mathbf{R}_{XY}$]).

One method for solving this set of simultaneous equations is to find what is known as the inverse of [$\mathbf{R}_{XX}$] which is symbolized in matrix algebra as [$\mathbf{R}^{-1}_{XX}$]. A technique for finding the inverse of [$\mathbf{R}_{XX}$] is to augment the predictor matrix, [$\mathbf{R}_{XX}$], with two $\mathbf{m}^x\mathbf{m}$ identity matrices and an $\mathbf{m}^x\mathbf{m}$ zero matrix. The **m** diagonal factors (based on the **X** variables) are found and removed from the augmented matrix. The original augmented matrix, the predictor-based diagonal factors, and the residual matrix would be as shown below.

augmented $\mathbf{R}_{XX}$ matrix		diagonal X factors		residual matrix
[$\mathbf{R}_{XX}$] [$\mathbf{I}_{mm}$]	DFA	[$\mathbf{D}_{XP}$]	and	[$\mathbf{0}_{mm}$] [$\mathbf{0}_{mm}$]
[$\mathbf{I}_{mm}$] [$\mathbf{0}_{mm}$]	yields	[$\mathbf{D}_{IP}$]		[$\mathbf{0}_{mm}$] [$\mathbf{R}^{-1}_{XX}$] ,

where:

[$\mathbf{R}_{XX}$] = the $\mathbf{m}^x\mathbf{m}$ correlations among the **m** predictor variables,
[$\mathbf{I}_{mm}$] = an $\mathbf{m}^x\mathbf{m}$ identity matrix,
[$\mathbf{0}_{mm}$] = an $\mathbf{m}^x\mathbf{m}$ zero matrix,

[$\mathbf{D}_{XP}$] = the predictors' correlations with (loadings on) the **m** diagonal factors,
[$\mathbf{D}_{IP}$] = the identity vectors' loadings on the **m** diagonal factors, and

[$\mathbf{R}^{-1}_{XX}$] = the $\mathbf{m}^x\mathbf{m}$ inverse of matrix [$\mathbf{R}_{XX}$].

If one multiplies the [$\mathbf{R}^{-1}_{XX}$] matrix by the [$\mathbf{R}_{XY}$] matrix, then one obtains the **z**-score weights for the multiple correlation prediction equation. These **z**-score weights are traditionally referred to as β-weights. That is, in matrix algebra,

$$[\ \beta_{YX}\ ] = [\ \mathbf{R}^{-1}_{XX}\ ] \times [\ \mathbf{R}_{XY}\ ]\ ,$$

where:

[β_{YX}] = the β-weights for the **m** predictors to predict the **n** criteria,
[$\mathbf{R}^{-1}_{XX}$] = the $\mathbf{m}^x\mathbf{m}$ inverse of matrix [$\mathbf{R}_{XX}$], and
[$\mathbf{R}_{XY}$] = the correlations of the **m** predictors with the **n** criteria.

2.3.3 Direct Calculation of β-Weights by Matrix Inversion

In the above subsection, it was shown that DFA is part of the procedure for calculating the **z**-score β-weights for either the MC and the MMC methods. An interesting variation of this procedure is to augment the combined predictor and criterion correlation matrix with identity and zero matrices as shown below and then perform the diagonal factor analysis based on the

predictor (i.e., **X**) variables. As shown below, the residual matrix, when this is done, yields the inverse, the β-weights and the unexplained variance and covariance of the criterion (i.e., **Y**) variables (symbolized here as [$\mathbf{U}_{YY}$]).

augmented correlation matrix				diagonal factors	the residual matrix		
[$\mathbf{R}_{XX}$]	[$-\mathbf{R}_{XY}$]	[$\mathbf{I}_{mm}$]		[$\mathbf{D}_{XP}$]	[$\mathbf{0}_{mm}$]	[$\mathbf{0}_{mn}$]	[$\mathbf{0}_{mm}$]
[$-\mathbf{R}_{YX}$]	[$\mathbf{R}_{YY}$]	[$\mathbf{0}_{nm}$]	yields	[$\mathbf{D}_{YP}$]	[$\mathbf{0}_{nm}$]	[$\mathbf{U}_{YY}$]	[β_{YX}]
[$\mathbf{I}_{mm}$]	[$\mathbf{0}'_{nm}$]	[$\mathbf{0}_{nn}$]		[$\mathbf{D}_{IP}$]	[$\mathbf{0}_{mm}$]	[β'_{YX}]	[$\mathbf{R}^{-1}_{XX}$]

where:

[$\mathbf{R}_{XX}$] = the $\mathbf{m}^x\mathbf{m}$ correlations among the **m** predictor variables,
[$-\mathbf{R}_{XY}$] = the reflected correlations of the **m** predictors with the **n** criteria,
[$-\mathbf{R}_{YX}$] = the transpose of matrix [$-\mathbf{R}_{XY}$],
[$\mathbf{R}_{YY}$] = the $\mathbf{n}^x\mathbf{n}$ correlations among the **n** criterion variables,

[$\mathbf{I}_{mm}$] = an $\mathbf{m}^x\mathbf{m}$ identity matrix,
[$\mathbf{0}_{nm}$] = an $\mathbf{n}^x\mathbf{m}$ zero matrix,
[$\mathbf{0}_{mn}$] = an $\mathbf{m}^x\mathbf{n}$ zero matrix,
[$\mathbf{0}_{nn}$] = an $\mathbf{n}^x\mathbf{n}$ zero matrix,

[$\mathbf{D}_{XP}$] = the predictors' correlations with (loadings on) the diagonal factors,
[$\mathbf{D}_{YP}$] = the criteria's correlations with (loadings on) the diagonal factors,
[$\mathbf{D}_{IP}$] = the identity vectors' loadings on the diagonal factors,

[$\mathbf{U}_{YY}$] = the $\mathbf{n}^x\mathbf{n}$ residual [$\mathbf{R}_{YY}$] matrix (i.e., unexplained **Y** correlations),
[β_{YX}] = the beta weights for the **m** predictors to predict the **n** criteria, and
[$\mathbf{R}^{-1}_{XX}$] = the $\mathbf{m}^x\mathbf{m}$ inverse of matrix [$\mathbf{R}_{XX}$].

Submatrix [$\mathbf{D}_{YP}$] contains the criterion variables' correlations with the independent predictor-based diagonal factors. When the entries in this submatrix are squared, they provide the incremental increase in criterion variance explained. The sums of these squared correlations across the factors for criterion **j** yield **j**'s variance that that can be explained by the predictors. This value is also the squared multiple correlation coefficient ($\mathbf{R}^2_{Y_j}$).

Of particular interest in having the inverse available is that the expected standard deviation of the **z**-score β-weight for predictor **i** to predict criterion variable **j** can be found by the equation:

$$s_{\beta Y_j X_i} = (- I_{Xi\,Xi}\ (1 - R^2_{Y_j.1\,...\,m}) / (N - m - 1))^{.5}\ .$$

where $\mathbf{I}_{Xi\,Xi}$ is the value found in the **i**th diagonal cell (i.e., row **i** and column **i**) of the inverse matrix $[\mathbf{R}^{-1}{}_{XX}]$ and $(\mathbf{1} - \mathbf{R}^2{}_{Y_j.1\,\ldots\,m})$ is the value found in the **j**th diagonal cell of the $[\mathbf{U}_{YY}]$ matrix. The **t**-test of significance (with **N-m-1** degrees of freedom) for each standard score beta weight is the ratio of the standard score beta weight divided by the standard deviation for that beta weight. That is,

$$t_{\beta Y_j X_i} = \beta_{YjXi} \;/\; s_{\beta Y_j X_i} \,.$$

While, on the surface, this appears to be totally different from traditional ANOVA methods for calculating sums of squares, mean squares, model coefficients, and testing their significance, the above calculations will arrive at mathematically identical solutions. This demonstrates that data analysis by ANOVA procedures is simply a special, but limited, case of the more general MC or MR methods.

It should be noted here that the square of the above equation will also yield an **F**-test (df = 1, N-m-1) for the standard score beta weight. That is,

$$t^2_{\beta Y_j X_i} = \beta^2_{YjXi} \;/\; I^2_{Xi\,Xi}\, s^2_{\beta Y_j\, X_i} \,.$$

$$F_{\beta Y_j X_i} = (\beta^2_{YjXi} \,/\, I^2_{Xi\,Xi}) \,/\, ((1 - R^2_{Y_j\,1\,\ldots\,m}) \,/\, (N - m - 1)) \,.$$

Finally, it should be noted that all of the equations in this subsection are equally applicable for one or more criterion variables. Indeed, the case in which only one criterion variable is to be analyzed is simply a special case of the more general situation in which several criteria are analyzed at the same time.

2.4 Multivariate Analysis of Variance (MANOVA)

Multivariate ANOVA (MANOVA) is typically accomplished when more than one criterion variable is to be analyzed. While the overall MANOVA tests of significance for main effects and interactions is somewhat different than in simple ANOVA, the model coefficients produced for each criterion are identical to those which would have been found had each

criterion been separately analyzed by ANOVA. It can also be shown that the same coefficients will be derived by the matrix inversion method just described in subsection 2.3.2.

2.5 Common Factor Analysis (FA) and Factor Rotation (FR)

Later, in discussing the Rotated Diagonal Factors (RDF) approach, our interest will be in meaningful independent factors that explain certain types of variance which causes the predictors to be related to the criterion variables. Before discussing the RDF method for doing this, it may be helpful to discuss briefly what is referred to as common factor analysis (FA) and factor rotation (FR).

2.5.1 Assumptions and Purpose of Common Factor Analysis

FA assumes that the total variance of any variable is composed of three unrelated kinds of variance: common, specific, and error. That is, for variable **i**,

$$\sigma^2_{i\ total} = \sigma^2_{i\ common} + \sigma^2_{i\ specific} + \sigma^2_{i\ error}\ .$$

The common part of the variance includes any part that causes variable i to be related to another variable in the correlation matrix. The specific variance is defined as non-error variance that is unrelated to other variables in that matrix. Thus, the reliable part of the total variance is the common plus the specific. The unique part of the variance is the specific plus the error.

FA further assumes that the common variance for any variable in a matrix can, itself, be composed of (i.e., explained by) **K** individual independent components. That is,

$$\sigma^2_{i\ common} = \sigma^2_{iA} + \sigma^2_{iB} + \ldots + \sigma^2_{ik} + \sigma^2_{iK}\ .$$

It has been shown that DFA can produce a set of **m** independent diagonal factors that can completely explain all of the relationships among the **m** variables on which those factors were based. However, these independent diagonal factors explain all the variance of the variables, not

just the common variance. Further, and more to the point, they are not particularly meaningful. In fact, the values found for these factors were dependent strictly on the sequence in which the diagonal factors were extracted. Traditional Factor Analysis (FA) techniques also analyze an $\mathbf{m}^{x}\mathbf{m}$ correlation matrix $[\mathbf{R}_{XX}]$, but for somewhat different purposes. Typically, the investigator is not particularly concerned with whether the variables contained in the matrix to be factored are predictors or criteria. The purpose of FA is to find a smaller set ($\mathbf{k} < \mathbf{m}$) of independent dimensions (i.e., factors) which can explain the correlations among all of the variables in the matrix. Usually, $\mathbf{K}$ is expected to be no more than half of what $\mathbf{m}$ is (i.e., $\mathbf{K}\ ^{3}\ \mathbf{m/2}$). That is, FA attempts to explain the common variance in a matrix in a parsimonious fashion. For two variables to be related to each other, they must share some variance in common. Thus, the set of factors with which FA is concerned are referred to as common factors. In fact, the goal of FA is to find an $\mathbf{m}^{x}\mathbf{k}$ factor matrix (i.e., $[\mathbf{F}_{XK}]$) such that when multiplied by its transpose will closely reproduce (i.e., explain) all of the off-diagonal entries of the $[\mathbf{R}_{XX}]$ matrix.

The entry ($\mathbf{f}_{ik}$) in matrix $[\mathbf{F}_{XK}]$ for the **i**th variable on factor **k** represents the relationship of that variable to that factor. These entries are traditionally referred to as factor loadings. The explained correlation between any two variable (e.g., **i** and **j**) is given by the sum of the products of the factor loadings of those two variables across all of the independent common factors. That is, the explained correlation between variables **i** and **j** (which is symbolized here as $\mathbf{r'}_{ij}$) will be:

$$r'_{ij} = \underset{k=1}{(f_{ik}{}^{x}f_{jk})} = f_{iA}{}^{x}f_{jA} + f_{iB}{}^{x}f_{jB} + \ldots + f_{ik}{}^{x}f_{jk} + \ldots + f_{1K}{}^{x}f_{1K}\ .$$

The amount of common variance explained for a variable by these independent factors is known as its communality and is, traditionally, symbolized as $\mathbf{h}^2$. That is, for variable **i**,

$$h^2_i = \sigma^2_{i\ common} = \underset{k=1\text{ to }K}{(f_{ik}\ {}^{x}\ f_{ik})} = \underset{k=1\text{ to }K}{(f^2_{ik})}.$$

The matrix which contains (a) the explained correlations in the off-diagonal cells and (b) the communalities in the main diagonal will be referred to as $[\mathbf{R}_{XX}]$. The purpose of FA, then, is to derive the factor matrix $[\mathbf{F}_{XK}]$ such that, in matrix algebra notation,

$$[\ \mathbf{F}_{XK}\ ]\ ^{x}\ [\ \mathbf{F'}_{XK}\ ] = [\ \underline{\mathbf{R}}_{XX}]\ .$$

where:

[$\mathbf{F}_{XK}$] = the $\mathbf{m}^x\mathbf{K}$ common factor matrix,
[$\mathbf{F}'_{XK}$] = the transpose of [$\mathbf{F}_{XK}$], and
[$\underline{\mathbf{R}}_{XX}$] = the explained **X** variable correlations and communalities.

Before going further, the term eigenvalue should be introduced and explained. It is the sum of the squares of all of the variables' loadings on a particular factor. Remembering that a factor loading is a correlation of a variable to an independent factor, and that the square of a correlation is the amount of that variable's variance explained by that factor, then it can be seen that the eigenvalue of a factor is the total amount of variance that is being explained by that factor across all variables. The eigenvalue of factor k is symbolized as $\mathbf{E}_k$ and is given by the equation:

$$E_k = \underset{i=1 \text{ to } m}{(f^2_{ik})} .$$

2.5.2 Extracting the Common Factors

There are several ways to accomplish FA, most of which use mathematical criteria to determine both (a) the initial locations of the independent factors and (b) when to stop extracting factors (i.e., when the residual correlations probably represent only chance relationships). The most frequently used FA technique are various modifications to the Principle Component method (Hotelling, 1933) which maximizes the total common variance being explained by each successive factor. These modifications, which require estimates of communalities instead of ones in the main diagonal of [$\mathbf{R}_{XX}$], are referred to as the Principle Factors or Principle Axes method. The necessity for communalities for the main diagonal entries (instead of ones) is because FA is only interested in explaining common variance, rather than all the variance, of each variable.

When FA is begun, the communality of each variable is first estimated by various techniques (e.g., the squared multiple correlation of that variable using all other variables as predictors, the highest absolute correlation in that variable's row or column, etc.) Even though these estimates may be incorrect, the FA proceeds and factors are extracted until a criterion for stopping the factor extraction is met. New estimates of communalities are then possible using the latest factor loadings obtained. The new estimates of communalities can be substituted for the old community estimates and the whole process can be repeated (i.e., iterated), until the communalities stabilize (i.e., the starting values of the communalities are virtually the same as the ending values).

2.5.3 Rotating the Common Factors to a Meaningful Set

Because various mathematical criteria were used in the factor extraction process, the final independent common factors will rarely represent dimensions which are particularly useful in understanding why the variables correlated the way they did. However, the extracted factors can usually be rotated to a more meaningful set of independent dimensions. By a meaningful set of factors, it is meant only that the rotated factors should have loadings (i.e., correlations of the variables with those factors) that are useful to the investigator in explaining the nature of the factors. For example, it is particularly useful if some variables have high loadings on some factors but do not load on any other factors. In that circumstance, the investigator may be able to identify what it is that those particular variables have in common that the other variables do not possess. In that way, the nature of that factor can be deduced. Factor Rotation (FR) of independent factors can be accomplished either graphically or mathematically and does not change the ability of the new set of factors to explain the correlations among the variables. The Varimax technique (Kaiser, 1959) is a frequently used rotation technique that uses mathematical criteria to search for a set of factors having what is referred to as simple structure (i.e., each factor has two or more variables with high loadings on it and all other variables have near-zero loadings on it).

2.6 Canonical Analysis (CA)

Before discussing the RDF method, a brief discussion of Canonical Analysis (CA) will be given because it has certain similarities to both RDF and to MANOVA. CA was also devised by Hotelling (Hotelling, 1935). Like FA, CA is concerned with common variance, but its interest is limited to explaining the common variance shared by two sets of variables, but not the additional common variance that may be shared by variables within either set. Of importance to this discussion is the conclusion (in the discussion of MC) that both subject effects and experimental effects, when coded as dichotomous variables, can be considered to be one set of variables (i.e., the predictor set), while multiple criteria can be considered as a second set of variables (i.e., the criterion set). CA is generally applicable to any situation in which one set of **X** variables is related to another set of **Y** variables, and the scores for both sets of variables are available for a common group of cases (e.g., individuals). As before, if the **m** variables in set **X** and the **n** variables in set **Y** are intercorrelated, then the overall intercorrelation matrix can be partitioned into four submatrices such that:

$$[\ \mathbf{R}\ ] = \begin{matrix} [\ \mathbf{R}_{XX}\ ] & [\ \mathbf{R}_{XY}\ ] \\ [\ \mathbf{R}_{YX}\ ] & [\ \mathbf{R}_{YY}\ ] \end{matrix} \quad ,$$

where submatrices $[\mathbf{R}_{XX}]$, $[\mathbf{R}_{XY}]$, $[\mathbf{R}_{YX}]$, and $[\mathbf{R}_{YY}]$ contain the same variables as discussed earlier.

2.6.1 Early Canonical Techniques

Hotelling's original canonical method related the two sets of scores by finding a pair of vectors (one from each set) that correlated higher than any other pair of vectors. The effects of the best pair of vectors were then removed (as were the diagonal factors in DFA and MC) from the [**R**] matrix. Then a second pair of vectors were obtained that, together, yielded the highest correlation and were orthogonal (i.e., unrelated) to the first pair of vectors. This procedure was continued until there were as many pairs of vectors as there were variables in the smaller set. This procedure was actually started by finding the β-weights (betas) for predicting the variables in one set from the variables in the other set. These β-weights for predicting set **Y** variables from

the set **X** variables were found by the equation which was discussed earlier. That is, $[\beta_{YX}] = [\mathbf{R}^{-1}_{XX}] \times [\mathbf{R}_{XY}]$. Similarly, the β-weights for predicting the **X** set variables from the **Y** set variables were found by a complementary equation. That is, $[\beta_{XY}] = [\mathbf{R}^{-1}_{YY}] \times [\mathbf{R}_{YX}]$. Next, the cross products of the two sets of β-weights were obtained, using either $[\mathbf{C}_{XX}] = [\beta_{XY}] \times [\beta_{YX}]$ (if **m** is smaller) or $[\mathbf{C}_{YY}] = [\beta_{YX}] \times [\beta_{XY}]$ (if **n** is smaller). Next, the eigenvalue (i.e., the proportion of variance explained by that pair of vectors) of the appropriate [**C**] matrix was computed. The square root of the eigenvalue was referred to as the canonical correlation $\mathbf{R}_{Ci}$, where **i** represents the **i**th pair of vectors found as explained above. $\mathbf{R}_{Ci}$ can be tested for significance using a χ^2 value based on the eigenvalue.

2.6.2 The Reduced Matrix Canonical (RMC) Approach

While the traditional Hotelling CA technique was useful in determining the amount and significance of overlapping common variance between two sets of variables, interpretation of the successive sets of weights for each of the independent canonical factors was rarely possible. Hotelling's original method did not obtain factor loadings, but simply the beta weights for the variables on those factors. Further, the β-weights were for independent factors that had never been rotated to meaningful positions. Under such conditions, interpreting the canonical factors was next to impossible. This author developed a modified approach (Wherry, Jr., 1975) for canonical correlation that permitted the obtaining of rotatable orthogonal factor loadings from the factor space shared by the two sets of variables. This approach, referred to as the Reduced Matrix Canonical (RMC) approach, was first used on a method he had developed (Wherry, Jr., 1965) for the **K**-Coefficient, a Pearson-type substitute for the Contingency Coefficient. The RMC approach had an added advantage over traditional CA at that time in that each successive extracted factor would explain the maximum amount of common criterion variance. When all possible canonical factors had been extracted, both approaches would explain the same amount of criterion variance across all variables. However, it is often the case that all canonical factors are not significant. For the same number of canonical factors, the RMC approach will always explain as much or more of the common criterion variance.

The approach used in the RMC approach was to, first, diagonally factor the larger set of variables. For discussion purposes, suppose there are more **X** than **Y** variables (i.e., $\mathbf{m} > \mathbf{n}$). When all **m** of the predictor-based diagonal factors have been removed from both matrices, the diagonal factors and residual correlation matrix will be as shown below.

original correlations			**diagonal factors**		**residual matrix**	
[$\mathbf{R}_{XX}$]	[$\mathbf{R}_{XY}$]	RDF	[$\mathbf{D}_{XM}$]	and	[$\mathbf{0}_{mm}$]	[$\mathbf{0}_{mn}$]
[$\mathbf{R}_{YX}$]	[$\mathbf{R}_{YY}$]	yields	[$\mathbf{D}_{YM}$]		[$\mathbf{0}_{nm}$]	[$\mathbf{U}_{YY}$] ,

where:

[$\mathbf{R}_{XX}$], [$\mathbf{R}_{XY}$], [$\mathbf{R}_{YX}$], and [$\mathbf{R}_{YY}$] are as discussed earlier,

[$\mathbf{D}_{XP}$] = the loadings of predictors on the predictor-based diagonal factors,
[$\mathbf{D}_{YP}$] = the loadings of criteria on the predictor-based diagonal factors,

[$\mathbf{0}_{mm}$] = an $\mathbf{m}^{x}\mathbf{m}$ zero matrix = residual correlations among the predictors,
[$\mathbf{0}_{mn}$] = an $\mathbf{m}^{x}\mathbf{n}$ zero matrix = residual correlations of criteria and predictors,
[$\mathbf{0}_{nm}$] = an $\mathbf{n}^{x}\mathbf{m}$ zero matrix = residual correlations of criteria and predictors,
[$\mathbf{U}_{YY}$] = the $\mathbf{n}^{x}\mathbf{n}$ residual correlations among the **n** **Y** variables.

Any remaining values in matrix [$\mathbf{U}_{YY}$] cannot be attributable to common variance shared by the two sets of variables because the diagonal factors already explain all of the variance in the larger **X** set. Thus, these residual correlations may be ignored. However, the loadings of the larger and smaller sets of variables on the predictor-based independent diagonal factors must explain all of the interrelationships among the two sets of variables, all of the variance among the larger **X** set, and some of the variance of the **Y** set. The correlations and communalities of the smaller **Y** set of variables (in the space occupied by the larger **X** set) may now be found by the equation

$$r_{YXiYXj} = \sum_{m=1} (d_{Yim} \times d_{Yjm}) \text{ , or, in matrix algebra terms,}$$

$$[\,\mathbf{R}_{YXYX}\,] = [\,\mathbf{D}_{YP}\,] \times [\,\mathbf{D}'_{YP}\,] \text{ ,}$$

where:

[$\mathbf{R}_{YXYX}$] = the criteria correlations explained by predictor-based diagonal factors,

[$\mathbf{D}_{YP}$] = the criteria correlations with the predictor-based diagonal factors, and

[$\mathbf{D}'_{YP}$] = the transpose of matrix [$\mathbf{D}_{YP}$].

Using this new [$\mathbf{R}_{YXYX}$] matrix, which contains the correlations of the **Y** variables in the **X** space, a traditional Principle Factors Analysis (PFA) is then accomplished. This yields independent common factors which explain as much of this criterion variance as possible with each successive factor. Subsequent steps in the RMC approach derive the loadings of the **X** variables on the derived common factors, and it is this set of factors which can be rotated to find a meaningful set. Again, the use of DFA played a central part in CA in the RMC approach.

2.7 The Rotated Diagonal Factor (RDF) Approach

In the Introduction (Section 1), it was stated that "in a multi-task, multi-criterion study, the total variance in performance on each criterion should be able to be divided into three types of independent variance components: (a) between-tasks, (b) within-tasks (but between criteria), and (c) within-criterion." The Rotated Diagonal Factors (RDF) approach, to be discussed now, permits accomplishment of this objective.

It has been shown that MANOVA and MMC techniques permit derivation of model coefficients for predicting any criterion variable. In both techniques, the analysis is concerned, in one way or another, with all of the criterion variance, some of which is attributable to main effects, some to interaction effects, and some to the error term. It has also been seen that both FA and CA can be used to derive independent dimensions of shared common variance, but neither of these latter techniques will necessarily account for all of the criterion variance. Thus, FA and CA do not accomplish what is needed.

The RDF approach, however, will derive independent dimensions that can both explain all of the variance of the criterion variables and all of the covariance of the criterion variables with the predictor variables. The RDF approach consists of four major steps:

1. obtaining the criterion-based diagonal factors,
2. rotating these factors to a meaningful structure,
3. using the predictor variables with MMC to predict the rotated diagonal factors, and
4. determining and testing the predictor B-weights (i.e., model coefficients).

These four steps are explained in more detail in the following subsections.

2.7.1 Step 1: Obtaining the Criterion-Based Diagonal Factors

The initial step in the RDF approach is to obtain a set of diagonal factors. However, unlike MC or MMC, this set of factors is based on diagonally factoring the criterion set of variables rather than the predictor set. Diagonally factoring the combined predictor and criterion matrix obtains loadings of both predictor and criterion variables on all **m** of the criterion-based diagonal factors. That is,

$$
\begin{array}{ccccc}
\textbf{original correlations} & & \textbf{diagonal factors} & & \textbf{the residual matrix} \\
[\ \mathbf{R}_{XX}\ ]\ [\ \mathbf{R}_{XY}\ ] & \text{DFA} & [\ \mathbf{D}_{XC}\ ] & & [\ \mathbf{U}_{XX}\ ]\ [\ \mathbf{0}_{XY}\ ] \\
[\ \mathbf{R}_{YX}\ ]\ [\ \mathbf{R}_{YY}\ ] & \text{yields} & [\ \mathbf{D}_{YC}\ ] & \text{and} & [\ \mathbf{0}_{YX}\ ]\ [\ \mathbf{0}_{YY}\ ]\ ,
\end{array}
$$

where:

$[\ \mathbf{R}_{XX}\ ]$ is the $\mathbf{m}^{x}\mathbf{m}$ correlation matrix of the predictor variables,
$[\ \mathbf{R}_{XY}\ ]$ is the $\mathbf{m}^{x}\mathbf{n}$ correlation matrix of the predictors with the criteria,
$[\ \mathbf{R}_{YX}\ ]$ is the $\mathbf{n}^{x}\mathbf{m}$ correlation matrix of the criteria with the predictors,
$[\ \mathbf{R}_{YY}\ ]$ is the $\mathbf{n}^{x}\mathbf{n}$ correlations matrix of the criterion variables,

$[\ \mathbf{D}_{XC}\ ]$ is the $\mathbf{m}^{x}\mathbf{n}$ criterion-based diagonal factor matrix (the **X** set loadings),
$[\ \mathbf{D}_{YC}\ ]$ is the $\mathbf{n}^{x}\mathbf{n}$ criterion-based diagonal factor matrix (the **Y** set loadings),

$[\ \mathbf{U}_{XX}\ ]$ is the $\mathbf{m}^{x}\mathbf{m}$ residual correlation matrix of the predictor variables,
$[\ \mathbf{0}_{XY}\ ]$ is an $\mathbf{m}^{x}\mathbf{n}$ zero matrix = residual correlations of predictors and criteria,
$[\ \mathbf{0}_{YX}\ ]$ is an $\mathbf{n}^{x}\mathbf{m}$ zero matrix = residual correlations of criteria and predictors,
$[\ \mathbf{0}_{XY}\ ]$ is an $\mathbf{n}^{x}\mathbf{n}$ zero matrix = residual correlations of the criterion variables.

These diagonal factors account for (i.e., explain) all of the criterion variance, all of the correlations among the criterion variables, and all of the correlations of the predictor variables with the criterion variables. Because of this property, the criterion variables' communalities across the **n** factors will all equal 1.0. Thus, like in MANOVA, all of the criterion variance is explained.

2.7.2 Step 2: Rotating the Criterion-Based Diagonal Factors

The second step in the RDF approach is to rotate these criterion-based diagonal factors to a meaningful structure. Because these diagonal factors are orthogonal (i.e., mathematically independent), they, like any set of common factors, can be rotated to find a more meaningful set of factors. To be successful, the rotation of the criterion-based diagonal factors must result in a structure that contains factors that can be associated with the three desired types mentioned earlier:

- between-tasks factors: significant loadings on criteria from more than one task,
- within-task factors: significant loadings on more than one criterion for one task but no significant loadings on criteria from any other task, and
- within-criterion factors: only one significant loading on one criterion of one task.

It should be noted here that the first two types of factors (i.e., between-tasks and within-task) do account for common criterion variance while the final type (i.e., within-criterion) only accounts for unique criterion variance. Thus, the RDF approach accounts for all of the criterion variance, but it also separates the common from the unique variance. Such a set of factors cannot possess simple structure and, because of this, the Varimax technique cannot be used to accomplish the needed rotations. However, the factors can be rotated graphically to find such factors. The final rotated factor loadings will be referred to as shown below.

diagonal factors		**rotated diagonal factors**
$[\ \mathbf{D}_{XC}\ ]$	factor rotation	$[\ \mathbf{F}_{XC}\ ]$
$[\ \mathbf{D}_{YC}\ ]$	yields	$[\ \mathbf{F}_{YC}\ ]$.

where:

$[\ \mathbf{D}_{XC}\ ]$ = predictors' correlations on unrotated criterion-based diagonal factors,
$[\ \mathbf{D}_{YC}\ ]$ = criteria's correlations on unrotated criterion-based diagonal factors,
$[\ \mathbf{F}_{XC}\ ]$ = predictor's correlations on the rotated diagonal factors, and
$[\ \mathbf{F}_{YC}\ ]$ = criteria's correlations on the rotated diagonal factors.

2.7.3 Step 3: Predicting the RDFs Using Predictor Variables

The third step in the RDF approach is to employ the multi-criterion multiple correlation (MMC) technique and use the **X**-variables to predict the rotated diagonal factors. The correlations among the predictors, [$\mathbf{R}_{XX}$], are already known. The correlations of the predictors with the rotated diagonal factors, [$\mathbf{F}_{XC}$], were found in the previous step. The correlations among the rotated diagonal factors, [$\mathbf{R}_{CC}$], are, because they are independent factors, an $\mathbf{m}^x\mathbf{m}$ identity matrix. The method for augmenting a combined matrix of predictors and criteria for accomplishing the MMC was described earlier in subsection 2.3.3 which dealt with the direct calculation of β-weights by matrix inversion. Here, the criteria to be predicted are now the rotated diagonal factors. The initial augmented matrix, predictor-based diagonal factors, and residual matrix after diagonally factoring are:

the initial augmented matrix			**diagonal factors**	**the final residual matrix**		
[$\mathbf{R}_{XX}$]	[$-\mathbf{F}_{XC}$]	[$\mathbf{I}_{mm}$]	[$\mathbf{D}_{XP}$]	[$\mathbf{0}_{mm}$]	[$\mathbf{0}_{mn}$]	[$\mathbf{0}_{mm}$]
[$-\mathbf{F}'_{XC}$]	[$\mathbf{R}_{CC}$]	[$\mathbf{0}_{nm}$]	[$\mathbf{D}_{CP}$]	[$\mathbf{0}_{nm}$]	[$\mathbf{U}_{CC}$]	[β_{CX}]
[$\mathbf{I}_{mm}$]	[$\mathbf{0}_{mn}$]	[$\mathbf{0}_{mm}$]	[$\mathbf{D}_{IP}$]	[$\mathbf{0}_{mm}$]	[β'_{CX}]	[$\mathbf{R}^{-1}_{XX}$],

where:

[$\mathbf{R}_{XX}$] = the original $\mathbf{m}^x\mathbf{m}$ correlations among the predictors,
[$-\mathbf{F}_{XC}$] = the reflected predictor correlations with the rotated diagonal factors,
[$-\mathbf{F}'_{XC}$] = the transpose of [$-\mathbf{F}_{XC}$],
[$\mathbf{R}_{CC}$] = the correlations of the rotated diagonal factors, an identity matrix,

[$\mathbf{I}_{mm}$] = an $\mathbf{m}^x\mathbf{m}$ identity matrix,
[$\mathbf{0}_{nm}$] = an $\mathbf{n}^x\mathbf{m}$ zero matrix,
[$\mathbf{0}_{mn}$] = an $\mathbf{m}^x\mathbf{n}$ zero matrix,
[$\mathbf{0}_{mm}$] = an $\mathbf{m}^x\mathbf{m}$ zero matrix,

[$\mathbf{D}_{XP}$] = the predictors' loadings on the predictor-based diagonal factors,
[$\mathbf{D}_{CP}$] = the RDFs' loadings on the predictor-based diagonal factors,
[$\mathbf{D}_{IP}$] = the identity vector loadings on the predictor-based diagonal factors,

[$\mathbf{U}_{CC}$] = the $\mathbf{n}^x\mathbf{n}$ unexplained variance of the rotated diagonal factors,
[β_{CX}] = the $\mathbf{n}^x\mathbf{m}$ betas to predict rotated diagonal factors using predictors,
[β'_{CX}] = the $\mathbf{m}^x\mathbf{n}$ transpose of [β_{CX}], and
[$\mathbf{R}^{-1}_{XX}$] = the $\mathbf{m}^x\mathbf{m}$ inverse of [$\mathbf{R}_{XX}$].

Submatrix [$\mathbf{D}_{CP}$] contains the correlations of the rotated diagonal factors with the predictor-based diagonal factors. When these entries are squared and summed across a given rotated diagonal factor, they show the total percentage of that variance of that RDF explained by the predictors. This is its squared multiple correlation coefficient. Each entry squared is the incremental increase in the rotated diagonal factors' variance explained by the successive predictor variables. If the separate sums of these entries are obtained for the predictor variables associated with each main effect and each interaction term, then the total variance explained by each effect can be shown in the "sums of squares" column of a "source table". When those values are divided by their appropriate degrees of freedom, the resulting values can be shown in the "mean squares" column, and the mean squares can then be used to test the significance of the various effects.

2.7.4 Step 4: Obtaining the **B**-Weights for the **X** Variables

The final step in the RDF approach is computation of the raw-score weights to apply to the **X** variables for predicting the rotated diagonal factors. Submatrix [β_{CX}] provides the **z**-score beta weights for the predictors. To convert from **z**-score β-weights to raw-score **B**-weights, the means and standard deviations of the rotated diagonal factors must be known. There are no actual raw scores for the rotated diagonal factors, and any desired values may be assigned for that purpose. By assigning means of zero and standard deviations of one for each RDF, the derived **B**-weights take on interesting properties. It should be remembered that all of the predictor variables are dichotomous variables with raw scores of "1" (if that level was applicable) or "0" (if that level was inapplicable). Thus, while **B**-weights for all predictor variables may be available, many of them will be multiplied by zero for the prediction of a specific data case. Indeed, the only **B**-weights that will contribute to the prediction of a specific data case will be, at most, one from each level of main effect or interaction. Because of this property, each **B**-weight describes, in terms of standard deviation units, the impact of that specific level of that specific effect for that RDF.

In this section, various multivariate analyses have been discussed and compared. The purposes, strengths, and weaknesses of these different techniques have also been discussed in relationship to the RDF approach. It has been shown that the RDF approach incorporates and combines logic from Multivariate Multiple Correlation (MMC), Multivariate Analysis of Variance (MANOVA), Factor Analysis (FA), and Factor Rotation (FR) to provide a new and different approach to the analysis of studies involving multiple tasks and multiple criteria for each task. In the next section, a practical example of the use of the RDF approach will be demonstrated.

3. DEMONSTRATION OF THE RDF APPROACH

3.1 The Experimental Study

The experimental study used to demonstrate the RDF approach was conducted at the Naval Air Warfare Center in Warminster, Pennsylvania, in 1992. Its major purpose was in identifying the impact of temporarily automating one of three typical ongoing Naval fighter pilot tasks. The three tasks included: (a) a continuous tracking (Trk) task, (b) a discrete tactical assessment (TA) task, and (c) a discrete communications (Com) task. These tasks are discussed below.

3.1.1 The Tracking Task

The tracking (Trk) task was a first-order compensatory-type tracking task in which Ss attempted to null out error in both elevation and azimuth by manipulation of a control stick. Four measures of performance used to evaluate tracking performance on this task included: (a) RMS Error in Elevation, (b) RMS Error in Azimuth, (c) Stick Manipulations (i.e., whenever S manipulated the stick in a different direction or changed significantly the amount of stick deviation) in Elevation, and (d) Stick Manipulations in Azimuth. Two additional tracking performance measures (i.e., Total RMS Error and Total Stick Manipulations) were available, but were not used in the RDF approach. The decision not to use these measures was based on the fact that unique variance in any of the first four measures would become common variance in a composite of those measures. Using both composite variables and their individual variables in the same intercorrelation matrix is never recommended if factor analysis is to be accomplished on that matrix since it necessarily confuses common and unique variance.

3.1.2 The Tactical Assessment Task

The tactical assessment task (TA) consisted of a binary classification (e.g., hostile or non-hostile) of new targets that appeared on the Ss' tactical screen at the time that they came within a certain range of the target. Ss reported their assessments of the target by pressing either a "hostile" or "non-hostile" button for the classification of the target. Measures of performance available for this task included: (a) Percent of Correct Responses, (b) Median Time for Correct Responses, (c) Percent of Incorrect Responses, (d) Median Time for Incorrect Responses, and (e) Percent of Misses (i.e., no response in a specified period of time). Since (e), the criterion measure of percent of misses, is completely predictable from knowing (a) and (c), its inclusion would not explain any additional criterion variance, therefore, it was not included in the analysis.

3.1.3 The Communication Task

A communication (Com) task consisted of verbal messages that contained three parts. Each message began with (a) a call sign followed by (b) a command to change a parameter (either altitude or heading) and, finally, (c) the value to which the specified parameter was to be changed. For example, the message communicated might be, "Bantam, change heading to 180 degrees." Ss were not to respond in any fashion unless the call sign was the specific one they had been assigned. This was included as part of the task because pilots typically hear commands directed to others, and must mentally screen out those types of messages. The method of response was either verbal (for some Ss) or manual (for all other Ss). If a S's mode of response was verbal, he had been instructed to respond either, "Roger, heading" or Roger, altitude." If the S's mode of response was manual, he had been instructed to manipulate a two-position switch mounted on the control stick to its first (altitude) or second (heading) position. Measures of performance on the Com task included: (a) Percent of Correct Responses, (b) Median Time for Correct Responses, (c) Percent of Incorrect Responses, (d) Median Time for Incorrect Responses, and (e) Percent of Misses (i.e., no response in a specified period of time). Results indicated no actual misses occurred, therefore, that measure was not used. Since there were no misses, measure (c) was completely predictable from knowing (a). Thus, measure (c) was also excluded for that reason.

3.1.4 Experimental Manipulations

Experimental manipulations, in addition to the assignment of various Ss to either the Verbal-Response or Manual-Response mode groups for the Com task, consisted of Conditions, Order (of conditions), and Period, each of which will be clarified below. The Order that Ss performed the Conditions was randomized by the investigator. Seven experimental conditions were investigated. There were seven performance sessions, each of which consisted of three consecutive performance periods. A short break followed the conclusion of each session (i.e., after Period 3 was finished). During both the first and third periods of these sessions, Ss performed all three of the tasks discussed above, and all three tasks were at what was judged to be an "easy" level of difficulty. Conditions referred to possible modifications to the tracking or tactical assessment tasks during period 2. Such modifications included:

- changing TA or Trk to a more difficult level, and/or
- automating TA or Trk so the Ss did not have to perform it.

The Com task was always at its "easy" level and was never automated. The seven experimental conditions were thus predicated on whether the tracking or tactical assessment tasks were "easy" or "hard" and whether they were automated or not during period 2. The seven different conditions presented to each S during some period 2 are as shown below.

Condition	Communications	Tracking	Tactical Assessment
1	easy	easy	easy
2	easy	easy	hard
3	easy	hard	easy
4	easy	easy	hard (automated)
5	easy	hard (automated)	easy
6	easy	easy (automated)	hard
7	easy	hard	easy (automated)

3.1.5 Subjects

Originally, an equal number of Ss were planned for the verbal-response and manual-response mode groups for the Com task. However, because of missing data for some conditions and some tasks, a total of eight Ss had complete data for the verbal-response Com mode group while six Ss had complete data for the manual-response Com mode group.

3.1.6 Creating the Dichotomous Predictor Variables

The main objective of this study was to determine the residual effects of the presented conditions. That is, the analysis attempted to determine if, having been exposed to the various conditions, the Ss would behave differently in Period 3 than they had behaved during Period 1. Because of this, only performance data from Period 1 and Period 3 were included for analysis. For each criterion, each of the 14 Ss had scores on the seven conditions for the two periods of interest (i.e., Period 1 and Period 3). Thus, the total number of data cases (**N**) for each criterion was 196 (= 14 x 7 x 2). The major experimental effects studied and their accompanying degrees of freedom are shown below:

Experimental and Individual Differences Effects		Degrees of Freedom	
G	2 Groups of Com task response-modes (Verbal or Manual)	1	= (2-1)
S/V	8 Ss within Group 1 (the verbal-response mode)	7	= (8-1)
S/M	6 Ss within Group 2 (the manual-response mode)	5	= (6-1)
O	7 Orders of performing the conditions	6	= (7-1)
P	2 Periods of performance (Period 1 and Period 3)	1	= (2-1)
C	7 Conditions	6	= (7-1)
PxC	2 Periods by 7 Conditions	6	= (2-1)(7-1)
PxO	2 Periods by 7 Orders	6	= (2-1)(7-1)
	Effects degrees of freedom	38	
	Residual degrees of freedom	157	
	Total degrees of freedom	195	= (N-1)

Because the order of conditions was assigned randomly, some combinations of condition and order (CxO) did not occur at all while other combinations of CxO occurred by chance alone six times. Because of missing data for some CxO cells and the widely different number of cases in other CxO cells, it was decided not to attempt analysis of the CxO interaction.

3.2 Data Analyses

Dichotomous predictor variables (coded as "1" if applicable to a data case and "0" if inapplicable) were created for each potential effect's degrees of freedom as discussed above. Data analyses following the procedures described in subsection 2.7 were carried out on a Macintosh SE computer using "The RDF/MMC Analysis Program" developed by the author (Wherry, Jr., 1994). The program can perform the RDF analysis or an MMC. Results from this program were checked for accuracy against commercially available statistical packages offering MMC analysis. It yielded identical results and tests of significance.

Table 1. Means and Standard Deviations of Predictor and Criterion Variables

Vb#	Variable Name	Mean	St.Dev.	Vb#	Variable Name	Mean	St.Dev.
Groups (of Subjects)				**Period x Condition Interactions**			
X01	Com Verbal-Response	0.5714	0.4949	X27	P 1 x C 1	0.0714	0.2575
Ss (in Verbal Response Group)				X28	P 1 x C 2	0.0714	0.2575
X02	Subject V1	0.0714	0.2575	X29	P 1 x C 3	0.0714	0.2575
X03	Subject V2	0.0714	0.2575	X30	P 1 x C 4	0.0714	0.2575
X04	Subject V3	0.0714	0.2575	X31	P 1 x C 5	0.0714	0.2575
X05	Subject V4	0.0714	0.2575	X32	P 1 x C 6	0.0714	0.2575
X06	Subject V5	0.0714	0.2575	**Period x Order Interactions**			
X07	Subject V6	0.0714	0.2575	X33	P 1 x O 1	0.0714	0.2575
X08	Subject V7	0.0714	0.2575	X34	P 1 x O 2	0.0714	0.2575
Ss (in Manual Response Group)				X35	P 1 x O 3	0.0714	0.2575
X09	Subject M1	0.0714	0.2575	X36	P 1 x O 4	0.0714	0.2575
X10	Subject M2	0.0714	0.2575	X37	P 1 x O 5	0.0714	0.2575
X11	Subject M3	0.0714	0.2575	X38	P 1 x O 6	0.0714	0.2575
X12	Subject M4	0.0714	0.2575	**Criterion Variables (Y-set)**			
X13	Subject M5	0.0714	0.2575	**Tactical Assessment Task (TA)**			
Order (of Conditions)				Y01	TA Percent Correct.	94.0978	7.9708
X14	Order 1	0.1429	0.3499	Y02	TA Md. RT Correct	1.4772	0.2117
X15	Order 2	0.1429	0.3499	Y03	TA Percent Incorrect.	2.2875	3.9264
X16	Order 3	0.1429	0.3499	Y04	TA Md. RT Incorrect	0.6970	0.9531
X17	Order 4	0.1429	0.3499	Y05	TA RMS Error Elev.	17.6668	9.4384
X18	Order 5	0.1429	0.3499	Y06	TA RMS Error Azim.	14.1387	6.8633
X19	Order 6	0.1429	0.3499	Y07	TA Stick Manip. Elev.	0.4868	0.1995
Periods (#1 vs #3)				Y08	TA Stick Manip. Azim.	0.7382	0.2414
X20	Period 1	0.5000	0.5000	**Communication (Com) Task**			
Conditions				Y09	Com Percent Correct	98.5624	5.1916
X21	Condition 1	0.1429	0.3499	Y10	Com Md. RT Correct	2.0814	0.6260
X22	Condition 2	0.1429	0.3499	Y11	Com Md. RT Incorrect	0.4430	1.4628
X23	Condition 3	0.1429	0.3499	Y12	Com RMS Error Elev.	12.9298	6.4181
X24	Condition 4	0.1429	0.3499	Y13	Com RMS Error Azim.	16.5623	9.3000
X25	Condition 5	0.1429	0.3499	Y14	Com Stick Manip. Elev.	0.5008	0.1830
X26	Condition 6	0.1429	0.3499	Y15	Com Stick Manip. Azim.	0.7295	0.1993

Table 1 shows the means and standard deviations for the 38 (**X**-set) predictor variables. Note that the mean of any predictor variable **i** is always the proportion ($\mathbf{p}_{=1}$) of cases that received a "1" on that variable while the standard deviation of that variable is the square root of $\mathbf{p}_{=1}(1-\mathbf{p}_{=1})$. For example, the $\mathbf{p}_{=1}$ for the Verbal-Response variable is .5714 (= 8/14 or 8 Ss out of 14 Ss), the $\mathbf{p}_{=1}$ for each Subject variable is .0714 (= 1/14 or 1 S out of 14 Ss); the $\mathbf{p}_{=1}$ value for each order and condition is .1429 (= 1/7), etc. Table 1 also shows the means and standard deviations of the 15 (**Y**-set) criteria investigated. TA (and Com) RMS and Stick Manipulation means show average tracking criterion performance prior to and immediately following the TA (or Com) tasks.

Table 2. [R_{XX}]: Original Correlations Among Predictor Variables

Vb	Variable	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19
01	Verb-Resp Mode	100	24	24	24	24	24	24	24	-32	-32	-32	-32	-32	00	00	00	00	00	00
02	Subject V1	24	100	-08	-08	-08	-08	-08	-08	-08	-08	-08	-08	-08	00	00	00	00	00	00
03	Subject V2	24	-08	100	-08	-08	-08	-08	-08	-08	-08	-08	-08	-08	00	00	00	00	00	00
04	Subject V3	24	-08	-08	100	-08	-08	-08	-08	-08	-08	-08	-08	-08	00	00	00	00	00	00
05	Subject V4	24	-08	-08	-08	100	-08	-08	-08	-08	-08	-08	-08	-08	00	00	00	00	00	00
06	Subject V5	24	-08	-08	-08	-08	100	-08	-08	-08	-08	-08	-08	-08	00	00	00	00	00	00
07	Subject V6	24	-08	-08	-08	-08	-08	100	-08	-08	-08	-08	-08	-08	00	00	00	00	00	00
08	Subject V7	24	-08	-08	-08	-08	-08	-08	100	-08	-08	-08	-08	-08	00	00	00	00	00	00
09	Subject M1	-32	-08	-08	-08	-08	-08	-08	-08	100	-08	-08	-08	-08	00	00	00	00	00	00
10	Subject M2	-32	-08	-08	-08	-08	-08	-08	-08	-08	100	-08	-08	-08	00	00	00	00	00	00
11	Subject M3	-32	-08	-08	-08	-08	-08	-08	-08	-08	-08	100	-08	-08	00	00	00	00	00	00
12	Subject M4	-32	-08	-08	-08	-08	-08	-08	-08	-08	-08	-08	100	-08	00	00	00	00	00	00
13	Subject M5	-32	-08	-08	-08	-08	-08	-08	-08	-08	-08	-08	-08	100	00	00	00	00	00	00
14	Order 1	00	00	00	00	00	00	00	00	00	00	00	00	00	100	-17	-17	-17	-17	-17
15	Order 2	00	00	00	00	00	00	00	00	00	00	00	00	00	-17	100	-17	-17	-17	-17
16	Order 3	00	00	00	00	00	00	00	00	00	00	00	00	00	-17	-17	100	-17	-17	-17
17	Order 4	00	00	00	00	00	00	00	00	00	00	00	00	00	-17	-17	-17	100	-17	-17
18	Order 5	00	00	00	00	00	00	00	00	00	00	00	00	00	-17	-17	-17	-17	100	-17
19	Order 6	00	00	00	00	00	00	00	00	00	00	00	00	00	-17	-17	-17	-17	-17	100
20	Period 1	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
21	Condition 1	00	00	00	00	00	00	00	00	00	00	00	00	00	00	-08	00	00	00	-08
22	Condition 2	00	00	00	00	00	00	00	00	00	00	00	00	00	-08	-08	25	00	08	00
23	Condition 3	00	00	00	00	00	00	00	00	00	00	00	00	00	08	-08	00	33	-08	-17
24	Condition 4	00	00	00	00	00	00	00	00	00	00	00	00	00	17	08	-08	08	-08	-08
25	Condition 5	00	00	00	00	00	00	00	00	00	00	00	00	00	08	42	-08	-17	00	-17
26	Condition 6	00	00	00	00	00	00	00	00	00	00	00	00	00	-08	-08	00	-08	25	17
27	P1 C1	00	00	00	00	00	00	00	00	00	00	00	00	00	00	-06	00	00	00	-06
28	P1 C2	00	00	00	00	00	00	00	00	00	00	00	00	00	-06	-06	17	00	06	00
29	P1 C3	00	00	00	00	00	00	00	00	00	00	00	00	00	06	-06	00	23	-06	-11
30	P1 C4	00	00	00	00	00	00	00	00	00	00	00	00	00	11	06	-06	06	-06	-06
31	P1 C5	00	00	00	00	00	00	00	00	00	00	00	00	00	06	28	-06	-11	00	-11
32	P1 C6	00	00	00	00	00	00	00	00	00	00	00	00	00	-06	-06	00	-06	17	11
33	P1 O1	00	00	00	00	00	00	00	00	00	00	00	00	00	68	-11	-11	-11	-11	-11
34	P1 O1	00	00	00	00	00	00	00	00	00	00	00	00	00	-11	68	-11	-11	-11	-11
35	P1 O1	00	00	00	00	00	00	00	00	00	00	00	00	00	-11	-11	68	-11	-11	-11
36	P1 O1	00	00	00	00	00	00	00	00	00	00	00	00	00	-11	-11	-11	68	-11	-11
37	P1 O5	00	00	00	00	00	00	00	00	00	00	00	00	00	-11	-11	-11	-11	68	-11
38	P1 O6	00	00	00	00	00	00	00	00	00	00	00	00	00	-11	-11	-11	-11	-11	68

Note: two decimal points omitted.

The intercorrelations of the predictors and criteria are shown in Tables 2, 3, and 4. Table 2 shows the intercorrelations among the predictors (i.e., [$\mathbf{R}_{XX}$]).

Table 2. (Cont.) [$\mathbf{R}_{XX}$]: Intercorrelations of Predictor Variables

Vb	Variable	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
01	Verb-Resp Mode	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
02	Subject V1	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
03	Subject V2	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
04	Subject V3	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
05	Subject V4	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
06	Subject V5	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
07	Subject V6	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
08	Subject V7	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
09	Subject M1	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
10	Subject M2	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
11	Subject M3	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
12	Subject M4	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
13	Subject M5	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
14	Order 1	00	00	-08	08	17	08	-08	00	-06	06	11	06	-06	68	-11	-11	-11	-11	-11
15	Order 2	00	-08	-08	-08	08	42	-08	-06	-06	-06	06	28	-06	-11	68	-11	-11	-11	-11
16	Order 3	00	00	25	00	-08	-08	00	00	17	00	-06	-06	00	-11	-11	68	-11	-11	-11
17	Order 4	00	00	00	33	08	-17	-08	00	00	23	06	-11	-06	-11	-11	-11	68	-11	-11
18	Order 5	00	00	08	-08	-08	00	25	00	06	-06	-06	00	17	-11	-11	-11	-11	68	-11
19	Order 6	00	-08	00	-17	-08	-17	17	-06	00	-11	-06	-11	11	-11	-11	-11	-11	-11	68
20	Period 1	100	00	00	00	00	00	00	28	28	28	28	28	28	28	28	28	28	28	28
21	Condition 1	00	100	-17	-17	-17	-17	-17	68	-11	-11	-11	-11	-11	00	-06	00	00	00	-06
22	Condition 2	00	-17	100	-17	-17	-17	-17	-11	68	-11	-11	-11	-11	-06	-06	17	00	06	00
23	Condition 3	00	-17	-17	100	-17	-17	-17	-11	-11	68	-11	-11	-11	06	-06	00	23	-06	-11
24	Condition 4	00	-17	-17	-17	100	-17	-17	-11	-11	-11	68	-11	-11	11	06	-06	06	-06	-06
25	Condition 5	00	-17	-17	-17	-17	100	-17	-11	-11	-11	-11	68	-11	06	28	-06	-11	00	-11
26	Condition 6	00	-17	-17	-17	-17	-17	100	-11	-11	-11	-11	-11	68	-06	-06	00	-06	17	11
27	P1 C1	28	68	-11	-11	-11	-11	-11	100	-08	-08	-08	-08	-08	08	00	08	08	08	00
28	P1 C2	28	-11	68	-11	-11	-11	-11	-08	100	-08	-08	-08	-08	00	00	31	08	15	08
29	P1 C3	28	-11	-11	68	-11	-11	-11	-08	-08	100	-08	-08	-08	15	00	08	39	00	-08
30	P1 C4	28	-11	-11	-11	68	-11	-11	-08	-08	-08	100	-08	-08	23	15	00	15	00	00
31	P1 C5	28	-11	-11	-11	-11	68	-11	-08	-08	-08	-08	100	-08	15	46	00	-08	08	-08
32	P1 C6	28	-11	-11	-11	-11	-11	68	-08	-08	-08	-08	-08	100	00	00	08	00	31	23
33	P1 O1	28	00	-06	06	11	06	-06	08	00	15	23	15	00	100	-08	-08	-08	-08	-08
34	P1 O2	28	-06	-06	-06	06	28	-06	00	00	00	15	46	00	-08	100	-08	-08	-08	-08
35	P1 O3	28	00	17	00	-06	-06	00	08	31	08	00	00	08	-08	-08	100	-08	-08	-08
36	P1 O4	28	00	00	23	06	-11	-06	08	08	39	15	-08	00	-08	-08	-08	100	-08	-08
37	P1 O5	28	00	06	-06	-06	00	17	08	15	00	00	08	31	-08	-08	-08	-08	100	-08
38	P1 O6	28	-06	00	-11	-06	-11	11	00	08	-08	00	-08	23	-08	-08	-08	-08	-08	100

Note: two decimal points omitted.

It may be noted in Table 2 that correlations of zero exist between various blocks of levels of main effects and various blocks of levels of the interaction terms. These blocks of zero correlations result because the design of the study caused these main effects and interactions to be independent of each other. For example, the Verbal-Response Mode and Subjects are independent of all other effects. Order, Period, and Conditions are also independent of each other.

Table 3 shows the correlations of the 38 predictors with the 15 criteria.

Table 3. [R_{XY}]: Original Correlations of Predictors With Criteria

		data predicated on TA stimuli								data predicated on Com stimuli						
		TA criteria				Tracking criteria				Com criteria			Tracking criteria			
Vb.	**Variable**	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Y8**	**Y9**	**Y10**	**Y11**	**Y12**	**Y13**	**Y14**	**Y15**
X01	Verb.-Resp. Mode	336	-390	-323	-234	010	011	105	058	-220	650	224	018	032	089	018
X02	Subject V1	040	-363	-009	-031	259	463	-098	-268	-203	234	175	496	216	-113	-263
X03	Subject V2	085	-064	-100	-037	-057	-035	189	126	-004	136	065	-020	-052	313	128
X04	Subject V3	010	-093	-085	-037	323	129	-120	-067	-092	206	061	114	350	-140	-021
X05	Subject V4	045	198	-039	-073	277	206	-129	-205	-073	216	139	214	294	-144	-204
X06	Subject V5	198	-178	-146	-169	-252	-220	-175	-064	-042	188	065	-221	-223	-217	-140
X07	Subject V6	107	-079	-055	-020	-162	-163	212	255	-162	-318	092	-176	-176	189	183
X08	Subject V7	023	-095	-070	-071	-150	-183	119	287	077	334	-084	-224	-138	051	239
X09	Subject M1	046	-040	024	098	028	038	-049	025	077	-247	-084	031	004	-015	-022
X10	Subject M2	-030	-095	084	093	-161	-149	043	041	077	156	-084	-187	-162	-025	024
X11	Subject M3	016	292	-146	-169	-043	-005	118	-079	077	-373	-084	001	-035	194	128
X12	Subject M4	-608	122	719	295	358	196	-425	-409	077	-056	-084	230	373	-390	-431
X13	Subject M5	-200	214	055	171	039	155	110	015	039	-360	-010	148	013	130	-039
X14	Order 1	-433	-045	226	289	236	189	-176	-005	-136	052	049	205	273	-069	001
X15	Order 2	019	047	-001	013	123	046	-044	-033	026	-004	041	037	095	-111	-005
X16	Order 3	073	006	-023	006	-042	-042	-046	-069	-059	-010	076	-044	-040	050	-112
X17	Order 4	016	021	001	-022	025	066	086	103	-000	-012	-014	064	033	097	120
X18	Order 5	077	130	-077	-186	-073	-073	025	-069	082	-016	-069	-034	-073	001	-070
X19	Order 6	145	-045	-068	-090	-153	-123	096	045	113	002	-124	-116	-148	-028	051
X20	Period 1	-035	-129	063	073	-038	-138	-028	-099	061	034	-059	-180	-070	-048	-108
X21	Condition 1	058	017	-047	-036	-001	039	-093	-138	001	033	-034	-046	-059	003	-019
X22	Condition 2	032	002	-031	-109	-097	-115	-059	-126	051	-032	-069	-054	-050	-022	-206
X23	Condition 3	-097	-118	133	101	288	379	071	157	-029	-011	087	308	256	154	226
X24	Condition 4	-093	087	047	090	-046	-140	006	069	062	-018	-069	-075	-002	019	049
X25	Condition 5	-119	015	-013	060	128	061	000	053	-217	031	167	038	089	-124	-048
X26	Condition 6	109	064	-066	-110	-158	-165	-024	-030	019	-006	041	-075	-104	-024	-005
X27	P1 x C1	047	-035	-009	-005	-008	-007	-033	-122	039	027	-010	-071	-057	014	-008
X28	P1 x C2	002	-035	-039	-065	-051	-068	051	-033	077	-026	-084	-043	-045	011	-088
X29	P1 x C3	-043	-137	068	-007	033	047	-075	-042	-004	-008	065	009	016	-111	008
X30	P1 x C4	-051	020	068	065	004	-127	015	064	077	-021	-084	-091	024	012	-028
X31	P1 x C5	-148	026	-009	106	118	069	-086	035	-181	048	092	009	091	-048	-011
X32	P1 x C6	062	000	007	016	-096	-119	-009	-035	034	029	-010	-064	-073	044	-013
X33	P1 x O1	-420	-043	205	242	177	064	-093	037	-105	065	018	055	190	-054	-020
X34	P1 x O2	-006	-012	007	029	087	035	-108	-045	034	-022	-010	-005	074	-076	005
X35	P1 x O3	055	-034	006	072	-049	-088	-047	-074	000	-006	065	-090	-054	054	-095
X36	P1 x O4	062	-010	-009	-061	-054	-047	000	-025	077	029	-084	-051	-067	-040	-005
X37	P1 x O5	085	068	-085	-111	-051	-063	033	-050	077	-000	-084	-046	-068	018	-027
X38	P1 x O6	085	-096	022	-022	-088	-085	094	-008	077	001	-084	-088	-084	-009	015

Note: three decimal points omitted.

Table 4 shows the original intercorrelations among the 15 criterion variables.

Table 4. [R_{YY}]: Original Correlations Among the Criteria

		data predicated on TA stimuli								**data predicated on Com stimuli**						
		TA criteria				**Tracking criteria**				**Com criteria**			**Tracking criteria**			
Vb.	**Variable**	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Y6**	**Y7**	**Y8**	**Y9**	**Y10**	**Y11**	**Y12**	**Y13**	**Y14**	**Y15**
Y1	TA Percent Correct	1000	-136	-721	-489	-529	-408	352	301	-004	083	-011	-431	-548	266	289
Y2	" Md. RT Correct	-136	1000	-012	-029	-012	-045	014	038	025	-370	-001	-031	011	057	071
Y3	" Percent Incorrect	-721	-012	1000	631	438	300	-368	-289	073	-042	-072	331	455	-305	-301
Y4	" Md. RT Incorrect	-489	-029	631	1000	243	173	-150	013	030	-047	-018	186	236	017	-024
Y5	" RMSE in Elev.	-529	-012	438	243	1000	854	-404	-421	-177	114	201	841	968	-321	-357
Y6	" RMSE in Azim.	-408	-045	300	173	854	1000	-261	-369	-240	088	251	950	805	-153	-285
Y7	" St. Manip. Elev.	352	014	-368	-150	-404	-261	1000	623	-003	-110	-068	-299	-450	687	601
Y8	" St. Manip. Azim.	301	038	-289	013	-421	-369	623	1000	008	-127	-058	-427	-427	549	769
Y9	Com Percent Correct	-004	025	073	030	-177	-240	-003	008	1000	-135	-826	-244	-172	051	097
Y10	" Md. RT Correct	083	-370	-042	-047	114	088	-110	-127	-135	1000	121	088	148	-139	-176
Y11	" Md. RT Incorrect	-011	-001	-072	-018	201	251	-068	-058	-826	121	1000	260	195	-088	-116
Y12	" RMS Error Elev.	-431	-031	331	186	841	950	-299	-427	-244	088	260	1000	827	-184	-332
Y13	" RMS Error Azim.	-548	011	455	236	968	805	-450	-427	-172	148	195	827	1000	-346	-379
Y14	" St. Manip. Elev.	266	057	-305	017	-321	-153	687	549	051	-139	-088	-184	-346	1000	596
Y15	" St. Manip. Azim.	289	071	-301	-024	-357	-285	601	769	097	-176	-116	-332	-379	596	1000

Note: three decimal points omitted.

3.2.1 Step 1: Obtaining the Criterion-Based Diagonal Factors

Step 1 of the RDF approach was accomplished. This consisted of obtaining the 15 Criterion-based diagonal factors. These unrotated diagonal factors (**D**$_{.C}$'s) explain all of the criterion variance, all of the correlations among the criteria, and all of the correlations of the criteria with the predictor variables.

Table 5. [D_{YC}]: Criterion Loadings on Criterion-Based Diagonal Factors

Vb.	Variable	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	h^2
Y01	TA Pct. Correct	1000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	1000
Y02	" Md.RT Correct	-136	991	000	000	000	000	000	000	000	000	000	000	000	000	000	1000
Y03	" Pct.Incorrect	-721	-111	684	000	000	000	000	000	000	000	000	000	000	000	000	1000
Y04	" Md.RT Incorrect	-489	-096	390	774	000	000	000	000	000	000	000	000	000	000	000	1000
Y06	" RMSE in Elev.	-529	-085	070	-067	839	000	000	000	000	000	000	000	000	000	000	1000
Y07	" RMSE in Azim.	-408	-101	-007	-043	748	512	000	000	000	000	000	000	000	000	000	1000
Y08	" St.Manip.Elev.	352	062	-156	116	-231	128	874	000	000	000	000	000	000	000	000	1000
Y09	" St.Manip.Azim.	301	079	-092	263	-276	-041	468	727	000	000	000	000	000	000	000	1000
Y10	Com Pct. Correct	-004	025	107	-014	-221	-144	-020	-050	957	000	000	000	000	000	000	1000
Y11	" Md.RT Correct	083	-362	-032	-037	151	-058	-086	-051	-107	901	000	000	000	000	000	1000
Y12	" Md.RT Incorrect	-011	-003	-118	029	244	126	-052	033	-774	003	554	000	000	000	000	1000
Y13	" RMSE in Elev.	-431	-091	015	-051	716	444	-028	-064	-028	-002	011	296	000	000	000	1000
Y14	" RMSE in Azim.	-548	-064	078	-089	788	-034	-050	018	-014	049	-003	098	211	000	000	1000
Y15	" St.Manip.Elev.	266	094	-151	278	-171	202	533	119	082	-027	-013	042	030	665	000	1000
Y16	" St.Manip.Azim.	289	112	-116	225	-204	011	457	458	105	-055	009	006	-022	126	590	1000

Note: three decimal points omitted.

Table 5 shows the correlations of the criterion variables with the 15 unrotated diagonal factors (i.e., submatrix [$\mathbf{D}_{YC}$]). It can be seen in Table 5 that the communality (i.e., the sum of the squares of the loadings of a variable across all factor), $\mathbf{h}^2$, of each criterion variable is equal to 1. This shows that these criterion-based diagonal factors do, in fact, explain all of the variance of each criterion.

Table 6 shows the predictors' loadings on (i.e., correlations with) the unrotated criterion-based diagonal factors (i.e., submatrix [$\mathbf{D}_{YC}$]).

Table 6. [D_{XC}]: Predictor Loadings on Criterion-Based Diagonal Factors

Vb.	Variable	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	h^2
X01	Verbal-Resp.Mode	336	-348	-175	-046	199	-077	048	014	-165	490	074	069	064	096	-007	605
X02	Subject V1	040	-360	-029	-045	296	428	-085	-141	-077	060	-023	215	-158	-079	-087	527
X03	Subject V2	085	-053	-066	032	-012	008	166	013	008	140	132	082	013	277	-032	165
X04	Subject V3	010	-093	-129	011	393	-335	-006	027	-038	095	-072	-066	185	-030	104	359
X05	Subject V4	045	206	024	-053	373	-071	-060	-124	-017	235	089	011	074	-040	-074	287
X06	Subject V5	198	-152	-030	-097	-196	-024	-310	002	-092	114	052	-050	027	-126	-048	254
X07	Subject V6	107	-065	023	023	-132	-050	177	141	-197	-367	-023	-035	027	070	-039	270
X08	Subject V7	023	-093	-093	-042	-170	-114	094	268	052	377	-005	-101	-027	-038	106	310
X09	Subject M1	046	-034	077	112	061	026	-061	052	094	-286	-049	004	-028	-041	-088	135
X10	Subject M2	-030	-100	075	051	-223	-003	017	-025	022	181	-007	-156	-009	-067	027	133
X11	Subject M3	016	297	-148	-096	-006	062	082	-184	088	-293	-044	008	117	167	161	327
X12	Subject M4	-608	040	417	-208	-003	-099	-128	-112	003	-008	-042	-029	-051	-013	-090	641
X13	Subject M5	-200	189	-100	169	-039	251	106	-059	076	-264	032	022	-020	-025	-176	302
X14	Order 1	-433	-105	-143	159	023	-018	-057	153	-114	041	-139	059	097	019	104	329
X15	Order 2	019	050	027	022	164	-122	001	-009	042	-014	094	-045	-087	-103	043	079
X16	Order 3	073	016	046	033	-004	-012	-078	-084	-075	-030	044	-041	022	132	-124	066
X17	Order 4	016	023	022	-026	038	089	096	103	026	011	-017	050	048	031	054	041
X18	Order 5	077	142	-010	-170	-037	-013	001	-097	065	032	001	108	-081	022	-075	096
X19	Order 6	145	-026	050	-053	-102	016	040	-033	091	008	-033	029	015	-134	023	069
X20	Period 1	-035	-135	033	038	-081	-201	001	-159	007	-017	-002	-231	-062	037	-059	157
X21	Condition 1	058	026	-002	-005	038	072	-131	-111	012	021	-083	-295	-141	114	111	180
X22	Condition 2	032	007	-010	-115	-103	-057	-087	-134	011	-042	-046	150	109	065	-204	138
X23	Condition 3	-097	-132	072	016	264	252	174	277	082	-040	110	-081	025	096	162	336
X24	Condition 4	-093	075	-016	076	-098	-183	027	030	017	033	-024	173	064	008	007	101
X25	Condition 5	-119	-001	-145	076	096	-111	054	071	-200	006	-031	-087	-144	-165	-074	172
X26	Condition 6	109	079	031	-079	-120	-051	-085	-055	-026	015	110	256	073	-000	032	138
X27	P1 x C1	047	-029	032	004	015	-002	-045	-147	032	004	034	-249	-102	096	100	122
X28	P1 x C2	002	-035	-061	-056	-062	-053	048	-087	061	-034	-027	062	004	-006	-138	053
X29	P1 x C3	-043	-144	030	-069	-011	039	-052	036	-001	-063	114	-125	-017	-065	110	083
X30	P1 x C4	-051	013	048	029	-028	-242	070	035	035	-008	-023	073	029	028	-124	096
X31	P1 x C5	-148	006	-169	129	072	-079	-056	099	-161	033	-129	-201	-061	-003	-004	179
X32	P1 x C6	062	009	077	022	-079	-063	-037	-083	-005	034	042	139	010	085	003	058
X33	P1 x O1	-420	-101	-158	114	-042	-161	039	124	-116	063	-128	-061	023	019	003	307
X34	P1 x O2	-006	-013	002	032	102	-085	-085	019	046	-051	012	-142	010	-005	079	058
X35	P1 x O3	055	-027	063	093	-024	-090	-068	-118	-031	-036	115	-065	002	144	-098	094
X36	P1 x O4	062	-002	053	-066	-035	004	-016	-032	064	036	-031	-019	-068	-040	021	027
X37	P1 x O5	085	080	-022	-069	-003	-042	009	-099	068	022	-038	039	-103	022	-028	051
X38	P1 x O6	085	-085	108	-040	-072	-013	086	-092	049	-019	-009	-024	051	-070	022	060

Note: three decimal points omitted.

3.2.2 Step 2: Rotating the Criterion-Based Diagonal Factors

Step 2 of the RDF approach requires the rotation of the criterion-based diagonal factors to a meaningful structure. The Varimax technique was attempted, but did not result in the desired meaningful set of rotated factors that clearly showed between-tasks, within-task, and within-

criterion factors discussed previously. Ultimately, a graphical rotation technique from "The RDF Analysis Program" was used. It did result in the desired type of factor structure. The results of step 2 of the RDF approach are shown in Table 4.

Table 7 shows the 15 criterion variables' loadings on (correlations with) the rotated criterion-based diagonal factors. It represents submatrix [$\mathbf{F}_{YC}$] discussed in subsection 2.7.2. Each of these loadings represent the relationship between a specific criterion variable and one of these independent factors.

Table 7. [$\mathbf{F}_{YC}$]: The Criterion Loadings on the RDFs

		Rotated Diagonal Factors															
Vb.	**Variable**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**h^2**
Y01	TA Pct. Correct	**552**	005	070	-003	**536**	-010	-104	**623**	051	025	-002	-002	000	002	008	1000
Y02	" Md. RT Correct	001	040	044	012	-028	-001	**369**	-064	**-924**	021	023	-005	-004	-010	010	1000
Y03	" Pct. Incorrect	**-469**	002	-110	-015	**-875**	024	-017	018	026	017	003	022	-001	-017	008	1000
Y04	" Md. RT Incorrect	**-266**	-003	136	047	**-611**	-019	-028	-017	030	**-728**	004	-039	-012	021	018	1000
Y05	" RMSE Elev.	**-997**	006	024	-039	031	-010	006	004	013	007	-036	019	012	026	-005	1000
Y06	" RMSE Azim.	**-846**	**-406**	056	098	099	-066	008	005	026	007	**-302**	023	-008	017	-005	1000
Y07	" St.Manip.Elev.	**429**	-034	**587**	199	129	-029	014	010	013	030	-018	**641**	016	017	018	1000
Y08	" St.Manip.Azim.	**454**	005	**835**	**-303**	-015	-055	-003	-002	-006	-034	006	000	003	-003	-009	1000
Y09	Com Pct. Correct	162	039	-017	002	-143	**974**	031	-003	-009	-009	026	-024	018	-004	011	1000
Y10	" Md.RT Correct	-117	025	-104	-025	141	-070	**-973**	-037	006	007	011	001	-006	-012	-001	1000
Y11	" Md.RT Incorrect	-195	-026	-014	-010	167	**-779**	-016	-000	-010	-017	-021	-016	**-569**	005	-005	1000
Y12	" RMSE Elev.	**-848**	**-503**	-006	123	073	-078	012	002	015	003	-012	025	-018	-013	-004	1000
Y13	" RMSE Azim.	**-976**	-009	-003	-055	006	-015	-028	-023	-020	020	048	-024	013	**-199**	-014	1000
Y14	" St.Manip. Elev.	**317**	-017	**712**	**620**	079	021	022	007	-015	-035	-006	001	013	-001	-009	1000
Y15	" St.Manip. Azim.	**376**	004	**709**	-048	059	050	065	006	-014	-017	004	026	001	007	**585**	1000
Factor Type		BT	WT	WT	WT	WT	WT	BT	WC	WC	WC	WC	WC	WC	WC	WC	

Note: three decimal points omitted.

Table 7 does show the type of structure desired. Factor 1, for example, is a between-tasks factor since it has high loadings on both the tactical assessment task (TA), the tracking task while accomplishing a TA task, and the tracking task while accomplishing a Com task. It even shows some non-zero loadings on the Com task. Factors 2, 3, and 4 are all within-task factors since they deal with multiple criteria for the tracking task (for both when the Ss performed the TA task and when they performed the Com task). Factor 5 is also a within-task factor since it has high loadings on more than one criterion for the TA task, but no other high loadings for any other tasks. Factor 6 is also a within-task factor since it has high loadings on more than one criterion

for the Com task, but no high loadings on criteria for any other task. Factor 7 is a between-tasks factor since it has high loadings on criteria for both TA and Com tasks. Factors 8 through 15 are all within-criterion factors since only one criterion from each task has high loadings on any of these factors. Table 8 shows the predictor loadings on the rotated diagonal factors.

Table 8. [F_{XC}]: The Predictor Loadings on the RDFs

		Rotated Diagonal Factors															
Vb.	**Variable**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**h^2**
X01	Verb-Resp Mode	-002	048	082	039	**377**	-149	**-610**	087	172	055	090	010	-073	-063	-016	605
X02	Subject V1	**-254**	**-487**	-148	095	170	-112	-180	109	**276**	-009	-051	053	009	142	-082	527
X03	Subject V2	050	019	213	**241**	062	005	-162	-014	022	046	089	-010	-130	-009	-038	165
X04	Subject V3	**-314**	**308**	040	-135	**271**	-015	-125	029	057	-060	084	-005	088	-160	093	359
X05	Subject V4	**-279**	069	-096	-007	207	003	-152	153	**-293**	-004	033	052	-086	-066	-068	287
X06	Subject V5	**253**	-004	**-274**	-160	071	-073	-174	022	105	038	-045	-157	-063	-041	-053	254
X07	Subject V6	173	095	200	-000	-058	-211	**294**	072	215	047	020	027	026	-015	-046	270
X08	Subject V7	161	101	192	-195	-020	060	**-376**	-176	-021	098	-043	-024	008	017	099	310
X09	Subject M1	-022	-047	023	-057	-019	074	**240**	144	124	-117	-024	-065	049	039	-085	135
X10	Subject M2	161	063	-053	-035	-171	034	-198	-072	030	-049	-126	061	004	011	033	133
X11	Subject M3	035	020	-004	**269**	140	077	**390**	-050	-152	094	-003	045	051	-110	156	327
X12	Subject M4	**-383**	092	**-292**	-019	**-583**	046	054	-090	-095	162	036	-013	044	038	-086	641
X13	Subject M5	-039	-203	078	117	-069	039	**349**	-158	-092	-163	-101	102	-039	034	-169	302
X14	Order 1	**-244**	-028	086	-066	-149	-128	-030	**-367**	068	-150	042	-136	134	-101	087	329
X15	Order 2	-116	098	-015	-114	068	053	026	086	-047	-047	019	052	-086	095	053	079
X16	Order 3	036	071	-061	131	008	-069	014	090	-007	-031	-011	-077	-049	-008	-125	066
X17	Order 4	-023	-090	142	005	-004	009	-004	032	-021	065	-012	012	017	-057	050	041
X18	Order 5	068	-040	-087	060	063	081	018	029	-138	156	111	041	002	066	-070	096
X19	Order 6	157	-058	-049	-074	011	096	-023	085	029	035	006	108	035	-027	029	069
X20	Period 1	040	**295**	-121	059	-076	037	-035	-028	132	-066	-069	054	009	095	-052	157
X21	Condition 1	007	111	-118	129	056	004	-022	048	-039	-006	**-268**	-103	078	167	115	180
X22	Condition 2	083	-011	-158	101	011	033	040	-006	012	079	160	-028	045	-119	-210	138
X23	Condition 3	**-276**	-151	**348**	-000	-041	030	-008	069	130	071	-222	-035	-110	-017	154	336
X24	Condition 4	033	058	047	-033	-080	036	004	-105	-073	-065	**240**	-011	028	-079	004	101
X25	Condition 5	-110	101	038	-190	060	-197	010	-148	-006	-093	-010	086	033	160	-070	172
X26	Condition 6	142	-084	-106	014	015	-006	003	054	-070	067	**245**	-038	-111	-108	030	138
X27	P1 x C1	011	155	-099	133	015	038	-025	056	022	-014	-187	-013	-033	128	108	122
X28	P1 x C2	049	011	-046	041	027	074	028	-056	052	035	084	079	031	-003	-137	053
X29	P1 x C3	-027	012	-060	-080	-049	-002	014	-011	152	056	-133	-018	-114	020	108	083
X30	P1 x C4	-009	162	065	-041	-078	059	008	-009	-003	-017	189	020	032	-027	-125	096
X31	P1 x C5	-105	159	051	-087	046	-169	-014	-191	-022	-130	-126	-091	125	089	-010	179
X32	P1 x C6	082	001	-053	097	-044	011	-044	067	-018	-016	163	-033	-043	-025	005	058
X33	P1 x O1	-178	155	096	-078	-161	-111	-050	**-401**	067	-102	031	-046	130	-010	-009	307
X34	P1 x O2	-083	133	-026	-056	041	048	040	037	026	-050	-087	-075	-008	007	078	058
X35	P1 x O3	040	148	-065	145	-025	-016	003	083	036	-093	014	-065	-116	017	-095	094
X36	P1 x O4	056	-011	-061	-022	-008	070	-039	057	-012	056	-019	024	032	062	026	027
X37	P1 x O5	050	015	-058	058	078	079	002	038	-077	056	067	040	042	100	-022	051
X38	P1 x O6	088	022	-050	006	-054	060	-020	099	090	027	-014	142	014	-052	028	060
	Factor Type	BT	WT	WT	WT	WT	WT	BT	WC	WC	WC	WC	WC	WC	WC	WC	

Note: three decimal points omitted; bold indicates five percent of the variance is explained.

3.2.3 Step 3: Predicting the RDFs Using Predictor Variables

Step 3 in the RDF approach is to find the β-weights for the predictor variables to predict the rotated diagonal factors. Subsection 2.7.3 had indicated that this can be accomplished by performing a predictor-based Diagonal Factor Analysis (DFA) on the following matrix:

augmented combined matrix		**diagonal factors**		**the residual augmented matrix**
$[\mathbf{R}_{XX}][-\mathbf{F}_{XC}][\ \mathbf{I}_{mm}]$		$[\mathbf{D}_{XP}]$		$[\mathbf{0}_{mm}][\mathbf{0}_{mn}][\mathbf{0}_{mm}]$
$[-\mathbf{F}_{XC}][\mathbf{R}_{CC}][\ \mathbf{0}_{nm}]$	yields	$[\mathbf{D}_{CP}]$	and	$[\mathbf{0}_{nm}][\mathbf{U}_{CC}][\beta_{CX}]$
$[\mathbf{I}'_{mm}][\mathbf{0}'_{nm}][\mathbf{0}_{mm}]$		$[\mathbf{D}_{IP}]$		$[\mathbf{0}_{mm}][\beta'_{CX}][\mathbf{R}^{-1}_{XX}]$.

Submatrix $[\mathbf{R}_{XX}]$ contains the intercorrelations among the **X**-set predictor variables which were previously computed and shown in Table 2. Submatrices $[-\mathbf{F}_{XC}]$ and $[-\mathbf{F}'_{XC}]$ contain the reflected correlations of the predictor variables with the rotated diagonal factors (RDFs), and were shown in Table 8. Submatrix $[\mathbf{R}_{CC}]$ contains the correlations among the 15 independent RDFs. Because these independent factors are unrelated to each other, this submatrix can be represented by an $\mathbf{n}^x\mathbf{n}$ identity submatrix (i.e., "1" in the main diagonal and "0" in all other cells). The identity submatrix (i.e., $[\mathbf{I}_{mm}]$), zero submatrix (i.e., $[\mathbf{0}_{nm}]$), and their transposes (i.e., $[\mathbf{I}'_{mm}]$ and $[\mathbf{0}'_{nm}]$), and $[\mathbf{0}_{mm}]$ were needed to augment the combined matrix. These submatrices were created and augmented to the combined matrix. The MMC technique was then accomplished by sequentially removing the predictor-based diagonal factors and removing their effects from the augmented combined matrix. When all of the predictor-based diagonal factors have been removed from the combined augmented matrix, it appears as shown above as the residual augmented matrix.

Extraction of the 38 predictor-based diagonal factors resulted in the submatrices described earlier including five zero matrices (i.e., $[\mathbf{0}_{mm}]$, $[\mathbf{0}_{mn}]$, $[\mathbf{0}_{mm}]$, $[\mathbf{0}_{nm}]$, and $[\mathbf{0}_{mm}]$). These submatrices show that all of the variance related to the predictor variables was removed from the combined augmented matrix; the predictor-based diagonal factors accounted for that variance. However, the residual matrix also contains four important submatrices: $[\ \mathbf{U}_{CC}\ ]$ (indicating the unexplained variance of the rotated diagonal factors), $[\ \beta_{CX}\ ]$ (the standard-score beta weights),

[β'_{CX}] the transpose of [β_{CX}], and [$\mathbf{R}^{-1}_{XX}$] (the inverse of matrix [$\mathbf{R}_{XX}$]). Together, as explained earlier, these matrices contain information needed to create the source table, compute the **B**-weights, and conduct the significance tests.

Table 9 shows the submatrix [$\mathbf{D}'_{CP}$]. It is the transpose of [$\mathbf{D}_{CP}$] and contains the relationships between the predictor-based diagonal factors (PDFs) and the rotated diagonal factors (RDFs). Both the PDFs and the RDFs are sets of independent factors. Thus, the communalities for the PDFs are the eigenvalues of the RDFs and the RDF communalities are the eigenvalues of the PDFs.

Table 9. [D'$_{CP}$]: The PDF Correlations on the RDFs

		Rotated Diagonal Factors															
	PDFs*	**F1**	**F2**	**F3**	**F4**	**F5**	**F6**	**F7**	**F8**	**F9**	**F10**	**F11**	**F12**	**F13**	**F14**	**F15**	**h^2**
01	Verb-Resp Mode	002	-048	-082	-039	-377	149	610	-087	-172	-055	-090	-010	073	063	016	605
02	Subject V1	261	514	173	-088	-082	078	034	-091	-242	023	075	-052	-027	-162	081	493
03	Subject V2	-015	066	-176	-254	018	-031	021	023	-015	-031	-059	005	113	-030	047	123
04	Subject V3	367	-227	-026	097	-199	-016	-014	-018	-054	075	-065	001	-096	124	-082	281
05	Subject V4	411	-020	115	-020	-175	-039	012	-156	310	032	-025	-060	070	051	072	354
06	Subject V5	-061	055	346	145	-075	035	040	-055	-042	-005	055	155	064	038	077	199
07	Subject V6	008	-038	-070	016	046	212	-496	-137	-189	-018	001	002	-014	023	100	366
08	Subject V7	031	-075	-110	294	028	-002	054	101	-006	-101	088	073	-000	-004	-024	143
09	Subject M1	024	033	-052	047	-107	-028	-047	-181	-189	105	-005	065	-027	-020	095	115
10	Subject M2	-168	-078	018	034	032	009	414	011	-130	055	104	-056	015	006	-011	243
11	Subject M3	-077	-052	-031	-295	-304	-036	-120	-011	034	-088	-003	-054	-032	142	-151	261
12	Subject M4	385	-159	296	-077	423	-014	227	032	-019	-204	-050	-008	-038	026	073	551
13	Subject M5	198	127	008	-318	032	-015	-008	148	-036	089	100	-171	048	051	239	290
14	Order 1	244	028	-086	066	149	128	030	367	-068	150	-042	136	-134	101	-087	329
15	Order 2	159	-095	000	127	-044	-032	-021	-025	036	073	-026	-030	065	-079	-068	078
16	Order 3	037	-088	048	-098	008	087	-014	-035	003	073	-001	097	041	010	100	057
17	Order 4	111	059	-154	008	024	029	002	017	019	-009	-000	030	-018	065	-057	051
18	Order 5	054	026	041	-051	-042	-041	-023	027	159	-115	-140	010	-008	-048	055	076
19	Order 6	-023	069	022	086	-005	-090	014	-026	057	-041	-105	-071	-058	039	-035	049
20	Period 1	-040	-295	121	-059	076	-037	035	028	-132	066	069	-054	-009	-095	052	157
21	Condition 1	008	-109	122	-109	-062	-021	022	-055	058	003	249	093	-083	-170	-127	173
22	Condition 2	-065	-005	165	-086	-009	-036	-034	035	-035	-060	-105	037	-068	098	162	100
23	Condition 3	240	117	-260	-034	006	-077	002	-127	-144	-097	276	035	113	-029	-139	302
24	Condition 4	-043	-049	-052	-045	054	-100	-010	-000	039	-006	-136	026	003	049	-008	045
25	Condition 5	060	-088	-102	118	-049	184	-008	131	-036	032	075	-071	-070	-159	094	142
26	Condition 6	-065	023	011	-016	-017	112	-020	016	-014	-057	-209	108	125	080	-023	100
27	P1 x C1	007	003	-022	-043	003	-037	000	-046	-017	-012	-021	-062	130	017	-065	032
28	P1 x C2	029	095	-145	058	-060	-068	-016	058	-014	-002	005	-134	025	-074	-041	072
29	P1 x C3	-221	-030	367	151	-012	030	-048	080	-047	-042	-059	-024	091	-020	-047	236
30	P1 x C4	017	-051	-045	099	-011	-051	-039	-085	-034	-085	-089	-065	036	-018	146	070
31	P1 x C5	023	-019	-050	038	-064	044	-012	134	060	037	117	203	-099	052	-069	106
32	P1 x C6	009	030	-074	-021	004	-021	028	-000	012	053	-021	077	-037	-042	-018	021
33	P1 x O1	047	-120	-127	047	067	048	040	213	022	-018	-030	-084	-048	-057	067	107
34	P1 x O2	000	007	-004	009	014	-014	-047	037	-068	-032	068	066	-019	096	-065	033
35	P1 x O3	006	-074	-016	-073	043	-010	-004	-023	-010	074	-050	082	111	041	007	042
36	P1 x O4	-004	022	039	-016	013	-088	072	-026	076	-008	027	046	-061	-094	-020	038
37	P1 x O5	000	007	-012	003	-067	-053	010	-021	045	040	-015	048	-057	-045	-069	024
38	P1 x O6	024	-000	-000	-028	032	-018	-009	-043	-022	-014	-001	025	-005	093	-114	028
h^2	= Communality	827	567	668	517	575	206	884	393	383	185	346	239	176	227	297	6.490
	Factor Type	BT	WT	WT	WT	WT	WT	BT	WC	WC	WC	WC	WC	WC	WC	WC	

**PDF factors represent the predictors named with all of the previous predictors' variance removed.*
Note: three decimal points omitted.

Table 10 shows the squares of these values. They represent the proportion of variance of the RDFs that can be explained by each PDFs, or conversely, the proportion of variance of the PDFs that can be explained by each RDF. They also represent the Multiple $\mathbf{R}^2$s of the RDFs.

Table 10. The PDF Squared Correlations on the RDFs

		Rotated Diagonal Factors															
	PDFs*	**F1**	**F2**	**F3**	**F4**	**F5**	**F6**	**F7**	**F8**	**F9**	**F10**	**F11**	**F12**	**F13**	**F14**	**F15**	**h^2**
01	Verb-Resp Mode	000	002	007	002	142	022	372	008	030	003	008	000	005	004	000	605
02	Subject V1	068	264	030	008	007	006	001	008	058	000	006	003	000	026	006	493
03	Subject V2	000	004	031	064	000	000	000	000	000	000	004	000	013	000	002	123
04	Subject V3	134	051	000	009	040	000	000	000	003	006	004	000	009	015	007	281
05	Subject V4	169	000	013	000	031	001	000	024	096	001	000	004	005	003	005	354
06	Subject V5	004	003	120	021	006	001	002	003	002	000	003	024	004	001	006	199
07	Subject V6	000	001	005	000	002	045	246	019	036	000	000	000	000	000	010	366
08	Subject V7	000	006	012	086	000	000	003	010	000	010	008	005	000	000	000	143
09	Subject M1	000	001	003	002	012	000	002	033	036	011	000	004	000	000	009	115
10	Subject M2	028	006	000	001	001	000	172	000	017	003	011	003	000	000	000	243
11	Subject M3	006	003	000	087	093	001	014	000	001	008	000	003	001	020	023	261
12	Subject M4	148	025	088	006	179	000	052	001	000	042	003	000	001	000	005	551
13	Subject M5	039	016	000	101	001	000	000	022	001	008	010	029	002	003	057	290
14	Order 1	060	000	007	004	022	016	000	135	005	022	002	018	018	010	008	329
15	Order 2	025	009	000	016	002	001	000	000	001	005	000	000	004	006	005	078
16	Order 3	001	008	002	010	000	008	000	001	000	005	000	009	002	000	010	057
17	Order 4	012	003	024	000	000	000	000	000	000	000	000	000	000	004	003	051
18	Order 5	003	000	002	003	002	002	000	000	025	013	020	000	000	002	003	076
19	Order 6	000	005	000	007	000	008	000	000	003	002	011	005	003	001	001	049
20	Period 1	002	087	015	003	006	001	001	000	017	004	005	003	000	009	003	157
21	Condition 1	000	012	015	012	004	000	000	003	003	000	062	009	007	029	016	173
22	Condition 2	004	000	027	007	000	001	001	001	001	004	011	001	005	010	026	100
23	Condition 3	057	014	067	001	000	006	000	016	021	009	076	001	013	000	019	302
24	Condition 4	002	002	003	002	003	010	000	000	002	000	018	000	000	002	000	045
25	Condition 5	004	008	010	014	002	034	000	017	001	001	006	005	005	025	009	142
26	Condition 6	004	000	000	000	000	013	000	000	000	003	044	012	016	006	000	100
27	P1 x C1	000	000	000	002	000	001	000	002	000	000	000	004	017	000	004	032
28	P1 x C2	000	009	021	003	004	005	000	003	000	000	000	018	000	005	002	072
29	P1 x C3	049	000	135	023	000	000	002	006	002	002	003	000	008	000	002	236
30	P1 x C4	000	003	002	010	000	003	002	007	001	007	008	004	001	000	021	070
31	P1 x C5	000	000	002	001	004	002	000	018	004	001	014	041	010	003	005	106
32	P1 x C6	000	000	006	000	000	000	000	000	000	003	000	006	001	002	000	021
33	P1 x O1	002	014	016	002	004	002	002	045	000	000	000	007	002	003	004	107
34	P1 x O2	000	000	000	000	000	000	002	001	005	001	005	004	000	009	004	033
35	P1 x O3	000	005	000	005	002	000	000	000	000	005	002	007	012	002	000	042
36	P1 x O4	000	000	001	000	000	008	005	000	006	000	000	002	004	009	000	038
37	P1 x O5	000	000	000	000	004	003	000	000	002	002	000	002	003	002	005	024
38	P1 x O6	000	000	000	000	001	000	000	002	000	000	000	000	000	009	013	028
	Sums = Mul. R^2	827	567	668	517	575	206	884	393	383	185	346	239	176	227	297	6.490
	Factor Type	BT	WT	WT	WT	WT	WT	BT	WC	WC	WC	WC	WC	WC	WC	WC	

**PDF factors represent the predictors named with all of the previous predictors' variance removed.*
Note: three decimal points omitted.

Next, using the values in Table 10, the sums of the values within a block of predictor variable effects are now added together to obtain the "sums of squares" (SSs) for those effects. These SSs show the proportion of variance explained by the various main effects and interaction terms. The sum of the SSs for a given criterion-based RDF is the total explained variance (i.e., $\mathbf{R}^2$) for that factor, and 1 - $\mathbf{R}^2$ is the residual (i.e., error) variance for that factor. If the SSs are now divided by the number of levels within each effect (i.e., their degrees of freedom), then the "mean squares" (MSs) for each effect can be obtained. Finally, if the MSs are divided by the residual MS, then F-values for each effect are obtained. The SSs, MSs, F-values, and their significance levels are shown in Table 11.

Table 11. Source Table for the RDFs

Effect	dfs	F1 BT	F2 WT	F3 WT	F4 WT	F5 WT	F6 WT	F7 BT	F8 WC	F9 WC	F10 WC	F11 WC	F12 WC	F13 WC	F14 WC	F15 WC
SS Table																
Verb-Resp	1	000	002	007	002	142	022	372	008	030	003	008	000	005	004	000
Subjects	12	599	381	303	387	371	058	492	121	251	090	048	075	038	071	132
Orders	6	102	026	036	040	027	036	002	138	035	048	033	035	027	025	030
Periods	1	002	087	015	003	006	001	001	000	017	004	005	003	000	009	003
Conditions	6	071	036	123	037	010	064	002	038	028	017	217	029	045	074	071
P x C	6	050	014	166	040	008	012	005	037	008	013	026	074	038	011	035
P x O	6	003	020	018	009	012	013	009	050	014	009	009	023	022	034	027
Residual	157	173	433	332	483	425	794	116	607	617	815	654	761	824	773	703
Mul. R^2	38	827	567	668	517	575	206	884	393	383	185	346	239	176	227	297
Total Var.	195	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
MS Table		**F1**	**F2**	**F3**	**F4**	**F5**	**F6**	**F7**	**F8**	**F9**	**F10**	**F11**	**F12**	**F13**	**F14**	**F15**
Verb-Resp	1	000	002	007	002	142	022	372	008	030	003	008	000	005	004	000
Subjects	12	050	032	025	032	031	005	041	010	021	008	004	006	003	006	011
Orders	6	017	004	006	007	004	006	000	023	006	008	006	006	005	004	005
Periods	1	002	087	015	003	006	001	001	000	017	004	005	003	000	009	003
Conditions	6	012	006	020	006	002	011	000	006	005	003	036	005	007	012	012
P x C	6	008	002	028	007	001	002	000	006	001	002	004	012	006	002	006
P x O	6	000	003	003	001	002	002	002	008	002	001	001	004	004	006	004
Residual	157	001	003	002	003	003	005	000	004	004	005	004	005	005	005	004
F Table		**F1**	**F2**	**F3**	**F4**	**F5**	**F6**	**F7**	**F8**	**F9**	**F10**	**F11**	**F12**	**F13**	**F14**	**F15**
Verb-Resp	1	.000	.84	3.19	.50	**52.9**	4.45	**503.0**	1.97	**7.60**	.580	1.95	.021	1.02	.807	.061
Subjects	12	**45.8**	**11.6**	**12.0**	**10.6**	**11.5**	.952	**55.4**	**2.63**	**5.35**	1.46	.967	1.30	.601	1.21	**2.46**
Orders	6	**15.6**	1.59	2.83	2.19	1.65	1.18	.517	**6.00**	1.50	1.56	1.33	1.20	.881	.834	1.12
Periods	1	1.44	**31.7**	**7.01**	1.12	2.14	.269	1.69	.200	4.44	.847	1.15	.613	.013	1.84	.618
Conditions	6	**10.9**	2.21	**9.71**	1.99	.607	2.13	.521	1.62	1.21	.565	**8.72**	.994	1.44	2.51	2.67
P x C	6	**7.66**	.838	**13.2**	2.17	.487	.396	1.12	1.61	.319	.430	1.05	2.58	1.24	.371	1.30
P x O	6	.417	1.22	1.45	.460	.752	.460	2.04	2.17	.595	.273	.366	.798	.693	1.15	1.02

Note: three decimal points omitted in SSs Table and MSs tabled portions.

3.2.4 Step 4: Obtaining the **B**-Weights for the Predictor Variables

To convert **z**-score beta weights (i.e., β-weights) to raw score weights (i.e., **B**-weights), and to compute the constants to be added, the means and standard deviations of the RDFs' scores must be known. For this purpose, the means of RDFs were assumed to be zero and their standard

deviations were assumed to be 1. Since the dichotomous scores of all 38 predictors are either "1" (if applicable) or "0" (if not applicable), the resulting factor score **B**-weights will show the impact of predictor variables in terms of standard deviation units of the RDFs. Table 12 shows the β–weights to apply to the predictors' z-scores for predicting the RDFs' z-scores.

Table 12. [β'_{CX}]: The z-Score β-Weights for Predicting the RDFs

		Rotated Diagonal Factors (RDFs)														
Vb	**Variable**	**F1**	**F2**	**F3**	**F4**	**F5**	**F6**	**F7**	**F8**	**F9**	**F10**	**F11**	**F12**	**F13**	**F14**	**F15**
01	Verb-Resp	-082	015	-107	1.045	078	072	-1.132	-155	205	-031	073	415	-042	-175	-210
02	Subject V1	-427	-444	-166	-131	138	-157	109	127	285	038	-127	-028	001	122	-136
03	Subject V2	-145	026	169	005	038	-048	125	013	049	089	003	-087	-128	-019	-095
04	Subject V3	-483	294	009	-344	232	-067	160	053	081	-009	-002	-082	075	-159	027
05	Subject V4	-450	072	-118	-226	172	-050	135	168	-244	043	-049	-029	-087	-071	-123
06	Subject V5	043	004	-283	-368	046	-120	114	046	126	082	-122	-223	-066	-048	-109
07	Subject V6	-031	096	157	-219	-074	-249	549	093	228	090	-061	-052	017	-024	-102
08	Subject V7	-042	102	150	-400	-038	003	-073	-138	009	138	-120	-100	000	006	032
09	Subject M1	-254	-028	-063	272	003	054	-090	079	250	-078	-064	078	016	-065	-248
10	Subject M2	-084	074	-133	292	-138	016	-497	-121	163	-015	-159	195	-025	-091	-138
11	Subject M3	-201	035	-088	575	151	056	049	-101	-006	118	-045	180	018	-203	-024
12	Subject M4	-589	101	-355	307	-521	028	-263	-138	047	181	-008	126	012	-066	-249
13	Subject M5	-270	-172	-012	434	-043	021	011	-201	050	-121	-136	233	-065	-070	-326
14	Order 1	-242	-190	-132	-068	-061	-027	012	-133	-037	-143	029	-068	131	-116	045
15	Order 2	-202	021	-074	-100	049	147	-029	124	-215	-035	066	204	-049	173	-056
16	Order 3	-071	-022	-057	-056	037	-058	005	-011	-129	027	-073	137	120	099	-154
17	Order 4	-061	-043	045	-094	002	-081	062	-093	-087	010	072	169	052	-098	-096
18	Order 5	-074	-046	-073	-015	-019	115	-004	-033	-209	149	019	178	047	123	-173
19	Order 6	007	-056	011	-155	064	105	-044	-014	-108	017	-016	146	111	172	-073
20	Period 1	026	306	-213	170	-120	-074	-072	020	119	-027	-105	338	-017	185	-341
21	Condition 1	-037	148	-071	135	059	-106	-028	-002	-047	050	-154	-190	138	190	058
22	Condition 2	048	118	-199	223	-048	-107	014	043	050	106	255	-235	-027	-186	-136
23	Condition 3	-444	-103	671	202	-019	030	-075	151	083	112	-165	-088	-094	068	132
24	Condition 4	090	070	022	103	-054	-085	-027	-177	-056	-011	220	-083	-033	-022	147
25	Condition 5	-000	118	051	-115	018	-258	031	-085	130	077	185	194	-102	167	-119
26	Condition 6	098	013	-101	009	050	-144	054	-012	009	132	291	-116	-194	-167	091
27	P1 x C1	029	-042	050	-042	034	015	049	017	066	021	037	027	-205	-049	083
28	P1 x C2	010	-163	207	-217	115	011	069	-178	062	035	-017	161	-024	102	-028
29	P1 x C3	322	021	-460	-283	047	-148	147	-134	142	053	089	-036	-166	-068	000
30	P1 x C4	-027	021	126	-156	053	-013	090	130	068	075	115	-004	-063	-022	-251
31	P1 x C5	-029	-033	110	-043	094	-138	-002	-155	-099	-098	-134	-294	116	-041	022
32	P1 x C6	-000	-070	124	022	-036	-036	-035	-018	019	-068	033	-068	010	068	-083
33	P1 x O1	-096	185	188	-015	-105	058	-084	-256	-066	001	029	-026	140	038	112
34	P1 x O2	-026	-004	004	044	-018	143	044	022	061	010	-099	-236	085	-181	299
35	P1 x O3	-034	100	016	151	-053	126	-029	115	-038	-131	078	-222	-088	-079	183
36	P1 x O4	-018	-044	-058	056	002	222	-127	109	-152	-006	-034	-150	163	113	213
37	P1 x O5	-029	-012	024	027	090	122	-008	088	-061	-060	030	-120	115	-020	262
38	P1 x O6	-049	001	001	056	-065	037	019	086	045	028	003	-051	010	-187	231

Note: three decimal points omitted.

The **B**-weights for predicting each RDF (using the predictor variables) and their levels of significance are shown in Table 13. Conversion from β-weights to **B**-weights is based on the earlier discussed equation where $\mathbf{B}_{ij} = \beta_{ij}\ s_{Y_i} / s_{X_j}$. Here, s_{Y_i} is RDF **i**'s standard deviation (assumed to equal 1.0) and s_{X_j} is predictor **j**'s standard deviation (shown in Table 1).

Table 13. [B'_{CX}]: Raw-Score B-Weights for Predicting RDFs

	Rotated Diagonal Factors (RDFs)														
X Variables	**F1**	**F2**	**F3**	**F4**	**F5**	**F6**	**F7**	**F8**	**F9**	**F10**	**F11**	**F12**	**F13**	**F14**	**F15**
Verb-Resp Md.	-.169	.035	-.216	**2.118**	.165	.139	**-2.28**	-.313	.417	-.070	.145	.838	-.086	-.345	-.428
Subject V1	**-1.66-**	**1.723**	**-.647**	-.509	*.534*	-.606	**.418**	.495	**1.105**	.150	-.494	-.111	.007	.468	-.523
Subject V2	**-.564**	.098	**.654**	.015	.144	-.182	**.484**	.050	.186	.348	.012	-.340	-.495	-.078	-.362
Subject V3	**-1.88**	**1.14**	.033	**-1.34**	**.897**	-.256	**.618**	.207	.313	-.031	-.006	-.322	.292	-.622	.109
Subject V4	**-1.75**	.278	*-.457*	**-.879**	**.666**	-.192	**.521**	*.652*	**-.947**	.170	-.190	-.115	-.335	-.283	-.472
Subject V5	.168	.014	**-1.10**	**-1.43**	.176	-.464	**.442**	.182	.487	.321	-.472	**-.871**	-.251	-.193	-.419
Subject V6	-.119	.374	**.608**	**-.854**	-.287	**-.961**	**2.13**	.362	**.882**	.353	-.239	-.207	.070	-.099	-.392
Subject V7	-.164	.395	**.580**	**-1.56**	-.153	.017	*-.285*	-.535	.034	.538	-.463	-.390	.002	.018	.132
Subject M1	**-.993**	-.104	-.247	**1.06**	.017	.206	**-.349**	.306	**.971**	-.309	-.251	.301	.064	-.250	**-.962**
Subject M2	*-.332*	.294	*-.521*	**1.14**	*-.530*	.060	**-1.93**	-.470	*.632*	-.065	*-.621*	*.755*	-.097	-.349	-.534
Subject M3	**-.786**	.138	-.341	**2.24**	*.589*	.215	.193	-.392	-.023	.453	-.177	*.696*	.072	*-.786*	-.091
Subject M4	**-2.29**	.395	**-1.38**	**1.19**	**-2.02**	.103	**-1.02**	-.536	.183	*.697*	-.035	.489	.047	-.252	**-.964**
Subject M5	**-1.06**	**-.667**	-.048	**1.69**	-.163	.078	.043	**-.782**	.193	-.474	-.528	**.902**	-.251	-.266	**-1.26**
Order 1	**-.701**	-.531	-.386	-.179	-.178	-.086	.026	-.387	-.100	-.415	.068	-.201	.389	-.329	.125
Order 2	*-.589*	.069	-.212	-.270	.135	.411	-.087	.350	-.613	-.102	.179	.575	-.124	.499	-.164
Order 3	-.212	-.052	-.170	-.143	.103	-.172	.005	-.037	-.369	.070	-.219	.388	.360	.277	-.441
Order 4	-.183	-.111	.122	-.251	.003	-.240	.171	-.272	-.244	.026	.195	.475	.161	-.282	-.275
Order 5	-.221	-.120	-.215	-.026	-.058	.321	-.019	-.096	-.594	.417	.038	.506	.148	.351	-.498
Order 6	.011	-.152	.023	-.432	.183	.297	-.129	-.044	-.315	.046	-.061	.421	.327	.493	-.213
Period 1	.045	.613	-.438	.334	-.226	-.149	-.147	.036	.213	-.046	-.222	.693	-.041	.369	-.699
Condition 1	-.100	.414	-.206	.373	.184	-.296	-.077	-.005	-.157	.149	-.439	-.532	.381	.545	.164
Condition 2	.144	.330	-.570	.619	-.125	-.299	.052	.124	.122	.309	.734	-.661	-.093	-.528	-.397
Condition 3	**-1.26**	-.304	**1.92**	.563	-.040	.094	-.206	.437	.215	.331	-.472	-.235	-.287	.198	.369
Condition 4	.265	.194	.061	.278	-.138	-.239	-.071	-.501	-.184	-.023	.635	-.224	-.110	-.063	.414
Condition 5	.002	.330	.140	-.342	.069	-.731	.094	-.239	.349	.224	.533	.570	-.314	.473	-.344
Condition 6	.283	.026	-.291	.006	.158	-.405	.164	-.034	.008	.387	.837	-.321	-.573	-.477	.258
P1 x C1	.105	-.158	.199	-.144	.118	.046	.191	.075	.288	.066	.139	.082	-.772	-.195	.332
P1 x C2	.028	-.626	.805	-.821	.436	.031	.256	-.684	.270	.122	-.078	.604	-.068	.395	-.095
P1 x C3	**1.24**	.096	**-1.79**	-1.083	.164	-.592	.564	-.516	.583	.188	.339	-.167	-.616	-.266	.020
P1 x C4	-.114	.087	.492	-.586	.189	-.059	.344	.505	.295	.276	.431	-.044	-.216	-.083	-.964
P1 x C5	-.119	-.120	.434	-.150	.345	-.551	-.013	-.598	-.353	-.390	-.534	-1.170	.488	-.155	.095
P1 x C6	-.009	-.262	.483	.107	-.159	-.155	-.145	-.060	.099	-.282	.113	-.285	.068	.267	-.315
P1 x O1	-.358	.704	.742	-.070	-.406	.242	-.319	-.991	-.265	.011	.140	-.091	.518	.147	.442
P1 x O2	-.086	-.024	.020	.159	-.066	.573	.174	.084	.234	.042	-.361	-.909	.303	-.707	1.168
P1 x O3	-.116	.378	.075	.573	-.203	.502	-.098	.446	-.147	-.501	.330	-.863	-.367	-.297	.715
P1 x O4	-.056	-.184	-.215	.201	.011	.878	-.486	.423	-.597	-.018	-.110	-.573	.615	.442	.827
P1 x O5	-.097	-.054	.103	.087	.349	.491	-.018	.336	-.240	-.225	.144	-.466	.423	-.076	1.025
P1 x O6	-.180	-.004	.016	.207	-.254	.149	.079	.339	.185	.108	.034	-.207	.029	-.724	.909
Constant	1.239	-.373	.420	-1.352	-.052	.218	1.261	.340	-.388	-.250	-.056	-.698	.065	.129	.850

Notes: Bold = significance < .001; italics = significance < .01.

Submatrix [U_{CC}], which contains the RDFs variance that was unexplained by the PDFs, is shown in Table 14. Table 15 shows a portion of the inverse of [R_{XX}]. These submatrices are shown because they contain the additional information used in determining:

$$s_{\beta Y_j X_i} = (- I_{Xi\,Xi}\ (1 - R^2_{Y_j.1\,\ldots\,m}) / (N - m - 1)\ )^{.5}\ .$$

where $I_{Xi\ Xi}$ is the value found in the **i**th diagonal cell (i.e., row **i** and column **i**) of the inverse matrix [$R^{-1}{}_{XX}$] and $(1 - R^2_{Y_j.1\,\ldots\,m})$ is the value found in the **j**th diagonal cell of the [U_{YY}] matrix. The **t**-test of significance (with N-m-1 dfs) for each x is $t_{\beta Y_j X_i} = \beta_{\,YjXi}\ /\ s_{\beta Y_j X_i}$.

Table 14. [U_{CC}]: The RDF Variance Unexplained by the PDFs

Vb	Variable	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
39	RDF 1	173	-049	036	057	-050	-010	-045	-002	-025	-020	-057	031	050	-022	-046
40	RDF 2	-049	433	022	095	056	-005	062	064	084	-003	-046	025	-037	057	-038
41	RDF 3	036	022	332	-017	-090	007	-058	052	-030	045	047	-028	-002	015	-107
42	RDF 4	057	095	-017	483	-033	-047	-001	-042	009	-008	014	-068	042	042	063
43	RDF 5	-050	056	-090	-033	425	027	093	-167	001	046	-007	-017	030	036	-066
44	RDF 6	-010	-005	007	-047	027	794	009	-063	112	-031	007	-039	013	-004	-053
45	RDF 7	-045	062	-058	-001	093	009	116	-044	064	034	-005	018	-040	006	018
46	RDF 8	-002	064	052	-042	-167	-063	-044	607	-033	-044	017	-014	096	-027	011
47	RDF 9	-025	084	-030	009	001	112	064	-033	617	026	077	-001	012	-019	045
48	RDF 10	-020	-003	045	-008	046	-031	034	-044	026	815	-038	-019	033	016	-008
49	RDF 11	-057	-046	047	014	-007	007	-005	017	077	-038	654	-018	030	109	065
50	RDF 12	031	025	-028	-068	-017	-039	018	-014	-001	-019	-018	761	035	-049	077
51	RDF 13	050	-037	-002	042	030	013	-040	096	012	033	030	035	824	-010	-023
52	RDF 14	-022	057	015	042	036	-004	006	-027	-019	016	109	-049	-010	773	033
53	RDF 15	-046	-038	-107	063	-066	-053	018	011	045	-008	065	077	-023	033	703

Table 15. A Portion of Submatrix [R^{-1}_{XX}]

Vb	Variable	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15
01	Verb-Resp	**-6857**	1784	1784	1784	1784	1784	1784	1784	-1784	-1784	-1784	-1784	-1784	000	000
02	Subject V1	1784	**-1857**	-929	-929	-929	-929	-929	-929	000	000	000	000	000	000	000
03	Subject V2	1784	-929	**-1857**	-929	-929	-929	-929	-929	000	000	000	000	000	000	000
04	Subject V3	1784	-929	-929	**-1857**	-929	-929	-929	-929	000	000	000	000	000	000	000
05	Subject V4	1784	-929	-929	-929	**-1857**	-929	-929	-929	000	000	000	000	000	000	000
06	Subject V5	1784	-929	-929	-929	-929	**-1857**	-929	-929	000	000	000	000	000	000	000
07	Subject V6	1784	-929	-929	-929	-929	-929	**-1857**	-929	000	000	000	000	000	000	000
08	Subject V7	1784	-929	-929	-929	-929	-929	-929	**-1857**	000	000	000	000	000	000	000
09	Subject M1	-1784	000	000	000	000	000	000	000	**-1857**	-929	-929	-929	-929	000	000
10	Subject M2	-1784	000	000	000	000	000–	000	000	-929	**-1857**	-929	-929	-929	000	000
11	Subject M3	-1784	000	000	000	000	000	000	000	-929	-929	**-1857**	-929	-929	000	000
12	Subject M4	-1784	000	000	000	000	000	000	000	-929	-929	-929	**-1857**	-929	000	000
13	Subject M5	-1784	000	000	000	000	000	000	000	-929	-929	-929	-929	**-1857**	000	000
14	Order 1	000	000	000	000	000	000	000	000	000	000	000	000	000	**-1857**	-2814
15	Order 2	000	000	000	000	000	000	000	000	000	000	000	000	000	-2814	**-1857**

Note: three decimal points omitted; bold indicates diagonal entries.

Finally, it has been indicated that one of the purposes of the RDF approach is to be able to partition the variance into that which is attributable to various independent effects (such as individual differences and experimental manipulations) and that which is attributable to in between-tasks, within-task, and within-criterion effects.

Recalling that the RDFs are independent from one another, it is possible to sum the portions of the effects' variance for a specific type of factors (i.e., between-tasks, within-task, and within-criterion). The data shown in Table 16 were derived from Table 11 (i.e., "The Source Table for the RDFs"). The variance explained (i.e., sums of squares (SSs)) for each effect were summed separately for the three factor types. The SSs were then divided by their appropriate degrees of freedom to find the mean squares (MSs). Finally, the MSs were divided by the residual MS to compute **F**-values.

At the far right of Table 16, another second of showing the impact of the various effects on the different factor types is provided. Here, the SSs for a particular factor type were divided by "15" (i.e., the total amount of all criterion variance across the 15 RDFs). This shows, by factor type, the percent of all RDF variance attributable to the various effects. For example, the "Subjects" effect accounted for a total of 13.78 percent (i.e., = 7.27 + 1.00 + 5.51) of all RDF variance.

Table 16. Source Table by Factor Types

		between-tasks			within-task			within-criterion			Percent of all RDF σ^2			
Effect	**dfs**	**SSs**	**MSs**	**F**	**SSs**	**MSs**	**F**	**SSs**	**MSs**	**F**	**BT**	**WT**	**WC**	**Sum**
Verb-Resp	1	.372	.3720	**206.67**	.175	.1750	**11.29**	.058	.0580	1.58	2.48	1.17	.39	4.04
Subjects	12	1.091	.0909	**50.44**	.150	.0125	.81	.826	.0688	1.87	7.27	1.00	5.51	13.78
Orders	6	.104	.0173	**9.67**	.165	.0275	1.77	.371	.0618	1.68	.69	1.10	2.47	4.26
Periods	1	.003	.0030	1.67	.112	.1120	**7.23**	.041	.0410	1.12	.02	.75	.27	1.04
Conditions	6	.073	.0122	**6.78**	.270	.0691	<u>2.90</u>	.519	.0865	<u>2.36</u>	.49	1.80	3.46	5.75
P x C	6	.055	.0092	**5.11**	.240	.0400	<u>2.58</u>	.242	.0403	1.10	.37	1.60	1.61	3.58
P x O	6	.012	.0020	1.11	.172	.0287	.77	.188	.0313	.85	.08	1.15	1.25	2.48
Residual	157	.290	.0018		2.466	.0157		5.755	.0367		1.93	16.44	38.37	56.74
Explained	38	1.710			2.534			2.245			11.40	16.89	14.97	43.26
Total	195	2.000			5.000			8.000			13.33	33.33	53.33	100.00

*Notes: a bold **F** was significant p< .01; an underlined **F** was significant p< .05.*

The interpretation of each of the between-tasks, within-tasks, and within-criterion RDFs is discussed in Section 4.

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4. INTERPRETATION OF THE FACTORS

A major purpose of the RDF approach is to gain insight into how performance on multiple tasks and multiple criteria are effected by both individual differences as well as by experimental manipulations. Each RDF now represents an independent composite of criterion performance variance and may be one of three theoretical types: (a) a between-tasks type causing Ss performance on different tasks to covary together, (b) a within-task type causing Ss' performance on different criteria for a specific task to covary together (but not influencing performance on any other task), and (c) a within-criterion type causing Ss to perform differentially well on a single criterion of one task (but not influencing behavior on any other criterion for that task or any other task). The predictor and criterion loadings on each RDF show the relationship of that variable with that factor. The square of these loadings shows that variable's portion of variance, which is accounted for by that factor. Because all of the variance of all of the criteria are being explained by the 15 rotated diagonal factors, they provide an alternative and more meaningful explanation of the overall criterion variance.

In the following subsections, each RDF will be discussed, first, in terms of how the various criteria relate to each other on that factor. This interpretation establishes the type of factor it appears to be (i.e., between-tasks, within-task, or within-criterion). Interpretation of the nature of an RDF can be based on both the criteria significantly related to it (as seen in Table 7) and its significant raw-score beta weights (as seen in Table 13). The **B**-weights show us how individual differences and experimental manipulations significantly effect scores for each factor. With regard to individual differences, Table 13 shows that 11 of the 15 factors had at least one subject variable with a significant ($p < .001$) raw-score **B**-weight on it. Of the remaining four factors, three of them had at least one subject variable with a significant ($p < .01$) raw-score **B**-weight. Thus, only one factor (i.e., RDF 13) failed to show a significant impact due to individual differences.

4.1 Adjusted Weights for Individual and Group Differences

While the **B**-weights are useful in indicating the relative effects of levels of subject effects, there is another way to examine the effect of individual differences. Before proceeding to the interpretation of the factors, it is useful to develop what can be referred to as adjusted weights. The purpose of adjusting the **B**-weights is to examine how each level of every effect contributes to the predicted RDF scores. It may be recalled that there were fourteen Ss in this study and eight of them used the "Verbal Response" mode during the Com tasks and six of them used the "Manual Response" mode. One dichotomous variable was created to represent this group variable. It may also be recalled that only seven dichotomous variables were created to represent seven of the eight Ss in the verbal response group, and that only five dichotomous variables were created to represent five of the six Ss in the manual response group. The choice of which seven and which five were represented as variables was arbitrary. From a prediction standpoint, it does not affect the analysis. However, when the β-weights were converted to **B**-weights, there was also a constant to be added (see the last row in Table 13). Since neither Subject V8 nor Subject M6 were represented by a separate dichotomous predictor variable, their **B**-weights are effectively zero. However, Subject M6's predicted factor score on any RDF included the constant for that RDF. Further, Subject V8 was a member of the "Verbal-Response" group, and his predicted factor score for any RDF included the **B**-weight for Verbal-Response as well as the constant for that factor. Table 17 shows the result of the predicted factors scores for all Ss based only on their Subject **B**-weights, the Verbal-Response **B**-weight, and the constant for each RDF.

Table 17. Predicted RDF Factor Scores Based on Subject Bs, the Verbal-Response B (When Applicable), and the Constant

Variable	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
Subject V1	-593	-2062	-450	255	657	-249	-603	520	1115	-167	-413	036	-017	252	-108
Subject V2	503	-238	852	781	267	173	-538	077	199	031	092	-191	-519	-293	050
Subject V3	-809	804	228	-574	1021	101	-404	232	325	-351	074	-173	267	-837	523
Subject V4	-683	-058	-262	-113	790	165	-502	679	-936	-149	-110	032	-360	-498	-058
Subject V5	1235	-321	-904	-665	300	-109	-581	207	499	002	-391	-721	-277	-408	-004
Subject V6	946	036	805	-088	-165	-606	1106	387	895	035	-157	-058	044	-315	021
Subject V7	903	058	776	-791	-028	371	-1309	-507	044	219	-384	-242	-022	-199	544
Subject V8	1066	-338	194	763	121	359	-1025	027	010	-316	081	145	-022	-221	419
Subject M1	244	-476	167	-293	-025	421	912	646	567	-556	-315	-389	127	-120	-119
Subject M2	903	-080	-107	-213	-573	277	-667	-133	228	-311	-683	065	-035	-221	307
Subject M3	449	-235	070	883	549	432	1453	-053	-428	205	-239	007	134	-657	750
Subject M4	-1058	025	-969	-156	-2058	320	241	-198	-223	450	-099	-202	109	-123	-122
Subject M5	182	-1039	365	335	-205	295	1305	-443	-212	-722	-593	213	-190	-138	-422
Subject M6	1231	-369	410	-1349	-037	213	1262	339	-405	-252	-066	-693	063	133	843
Group Differences															
Mean V-R	321	-265	155	-054	370	026	-482	203	269	-087	-151	-147	-113	-315	173
Mean M-R	325	-362	-011	-132	-391	326	751	026	-079	-198	-333	-166	035	-188	206
Mn V - Mn M	-004	097	166	078	**762**	-301	**-1233**	176	**348**	111	182	020	-148	-127	-033

Note: Three decimal points omitted. Bold indicates significance p< .001.

Mean differences of the two groups for RDFs are attributable to either (a) differences in "capability" of Ss within each group or (b) differences attributable to the "mode of response" Ss used in the Com task. It has already been seen that RDFs 5, 7, and 9 were significantly different (p< .001 level) from what would be expected by chance alone (if assignment to those groups actually had no effect). This conclusion is also supported by the fact that those RDFs had the largest differences between the means of the two groups (as shown at the bottom of Table 17).

The two groups discussed above could be statistically equated by subtracting the mean of each group from the predicted scores for each S in that group. When this computation is performed, the average of the adjusted subject weights will be zero, and the difference between the means of the two groups (shown in Table 17) must now be used as the "Verb-Resp" effect. Finally, the mean of the manual-response group must be used as a "constant" to be added for each predicted factor score. The predictor adjusted weights for the subjects and group effects are shown in Table 18.

Table 18. Group and Subject Adjusted Weights for Predicting RDFs

Variable	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
Verb-Resp	-004	097	166	078	**762**	-301	**-1233**	176	**348**	111	182	020	-148	-127	-033
Subject V1	-914	-1797	-605	309	287	-275	-121	317	846	-080	-262	183	096	567	-281
Subject V2	182	027	697	835	-103	147	-056	-126	-070	118	243	-044	-406	022	-123
Subject V3	-1130	1069	073	-520	651	075	078	029	056	-264	225	-026	380	-522	350
Subject V4	-1004	207	-417	-059	420	139	-020	476	-1205	-062	041	179	-247	-183	-231
Subject V5	914	-056	-1059	-611	-070	-135	-099	004	230	089	-240	-574	-164	-093	-177
Subject V6	625	301	650	-034	-535	-632	1588	184	626	122	-006	089	157	000	-152
Subject V7	582	323	621	-737	-398	345	-827	-710	-225	306	-233	-095	091	116	371
Subject V8	745	-073	039	817	-249	333	-543	-176	-259	-229	232	292	091	094	246
Subject M1	-081	-114	178	-161	366	095	161	620	646	-358	018	-223	092	068	-325
Subject M2	578	282	-096	-081	-182	-049	-1418	-159	307	-113	-350	231	-070	-033	101
Subject M3	124	127	081	1015	940	106	702	-079	-349	403	094	173	099	-469	544
Subject M4	-1383	387	-958	-024	-1667	-006	-510	-224	-144	648	234	-036	074	065	-328
Subject M5	-143	-677	376	467	186	-031	554	-469	-133	-524	-260	379	-225	050	-628
Subject M6	906	-007	421	-1217	354	-113	511	313	-326	-054	267	-527	028	321	637
Constants	325	-362	-011	-132	-391	326	751	026	-079	-198	-333	-166	035	-188	206

Note: underlined S scores show max. and min. values within that group.

4.2 Adjusted Weights for Experimental Effects

In the preceding subsection, it was shown that, even though all Ss were not represented with separate predictor dichotomous variables, it was still possible to determine their relative position on the RDF dimension. Likewise, "Order 7," "Condition 7," "Period 3," and all cells representing interactions of these levels were not represented by separate predictor variables. As with the Ss, relative locations on those levels of main effects and interactions on each RDF can be found. First, zero values for **B**-weights for non-represented levels of main effects and interactions were assumed. Next, predicted RDF scores were developed by applying all the relevant B-weights (i.e., Order, Period, Condition, PxC, and PxO) for each cell. The mean of the predicted RDF score cell values was then subtracted from each predicted cell value. Means for each Order, Period, and Condition were found and subtracted from the predicted cell values.

Table 19 shows the adjusted weights found by a procedure similar to that discussed for Ss.

Table 19. Experimental Manipulation Adjusted Weights for Predicting RDFs

Variable	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
Order 1	**-180**	-065	-012	-028	-113	-077	-017	**-249**	057	-134	029	-133	112	-127	084
Order 2	-105	052	-046	-031	040	140	006	147	-057	-021	002	034	-096	052	076
Order 3	022	061	-023	067	011	-074	004	059	-021	-030	-049	-026	-013	029	-080
Order 4	040	-032	042	-019	003	-049	012	-026	-036	016	040	042	044	-072	-007
Order 5	021	-019	-035	046	026	097	005	023	-112	128	019	066	015	083	-059
Order 6	092	-028	047	-082	034	040	-025	034	040	033	-027	068	027	066	003
Order 7	110	033	026	047	000	-079	013	012	127	009	-015	-052	-089	-030	-017
Period 1	017	**149**	**-082**	056	-048	-009	-027	-003	063	-024	-043	096	-002	072	-099
Period 3	-017	**-149**	**082**	-056	048	009	027	003	-063	024	043	-096	002	-072	099
Condition 1	-009	095	-112	086	053	020	-021	043	-056	-006	-235	-087	104	156	096
Condition 2	066	-001	-152	084	-012	012	027	-017	037	050	149	-066	029	-126	-176
Condition 3	**-270**	-125	**375**	033	-019	075	-019	121	112	072	-220	-017	-109	025	129
Condition 4	090	048	019	-003	-051	027	000	-075	-064	-040	178	004	004	-042	018
Condition 5	-001	069	040	-164	042	-208	012	-126	038	-039	019	136	024	137	-111
Condition 6	111	-054	-105	-008	009	-043	019	016	-024	031	208	-061	-121	-142	046
Condition 7	013	-032	-066	-028	-023	119	-018	037	-042	-067	-099	089	068	-009	-003
P1 x C1	-007	-001	013	031	-005	030	002	034	015	010	009	029	-079	-026	063
P1 x C2	-017	-067	100	-059	037	023	008	-071	011	010	-015	095	012	067	-015
P1 x C3	**139**	030	**-242**	-090	001	-051	051	-042	053	026	035	-003	-059	-036	022
P1 x C4	-035	030	051	-026	004	016	023	090	016	037	048	013	-008	-013	-104
P1 x C5	-036	003	043	030	025	-046	-023	-052	-068	-049	-076	-132	082	-022	033
P1 x C6	-022	-015	050	063	-040	005	-040	016	-009	-034	007	-019	029	032	-020
P1 x C7	-022	020	-012	052	-022	023	-022	025	-018	-000	-009	015	024	-002	022
P3 x C1	007	001	-013	-031	005	-030	-002	-034	-015	-010	-009	-029	079	026	-063
P3 x C2	017	067	-100	059	-037	-023	-008	071	-011	-010	015	-095	-012	-067	015
P3 x C3	**-139**	-030	**242**	090	-001	051	-051	042	-053	-026	-035	003	059	036	-022
P3 x C4	035	-030	-051	026	-004	-016	-023	-090	-016	-037	-048	-013	008	013	104
P3 x C5	036	-003	-043	-030	-025	046	023	052	068	049	076	132	-082	022	-033
P3 x C6	022	015	-050	-063	040	-005	040	-016	009	034	-007	019	-029	-032	020
P3 x C7	022	-020	012	-052	022	-023	022	-025	018	000	009	-015	-024	002	-022
P1 x O1	-030	077	080	-030	-042	-021	-028	-139	-018	013	014	045	040	039	-034
P1 x O2	005	-017	-012	-000	001	022	036	000	046	017	-050	-060	012	-071	060
P1 x O3	001	035	-006	053	-016	013	-001	047	-004	-053	038	-053	-074	-020	002
P1 x O4	009	-037	-043	006	011	061	-050	044	-061	009	-018	-017	051	076	017
P1 x O5	003	-021	-002	-009	055	011	010	033	-015	-018	014	-002	027	010	041
P1 x O6	-007	-020	-004	003	-020	-037	019	025	036	019	003	031	-025	-055	003
P1 x O7	018	-015	-014	-022	010	-050	014	-011	015	012	-001	058	-030	020	-090
P3 x O1	030	-077	-080	030	042	021	028	139	018	-013	-014	-045	-040	-039	034
P3 x O2	-005	017	012	000	-001	-022	-036	-000	-046	-017	050	060	-012	071	-060
P3 x O3	-001	-035	006	-053	016	-013	001	-047	004	053	-038	053	074	020	-002
P3 x O4	-009	037	043	-006	-011	-061	050	-044	061	-009	018	017	-051	-076	-017
P3 x O5	-003	021	002	009	-055	-011	-010	-033	015	018	-014	002	-027	-010	-041
P3 x O6	007	020	004	-003	020	037	-019	-025	-036	-019	-003	-031	025	055	-003
P3 x O7	-018	015	014	022	-010	050	-014	011	-015	-012	001	-058	030	-020	090
Constants	-110	153	-068	066	-037	-076	-031	-038	-025	046	061	131	012	129	-146

Note: Three decimal points omitted for all values. Bold indicates significance $p < .001$.

In Table 19, the means of the adjusted weights for the seven Orders, two Periods, seven Conditions, and all interactions are now zero. Thus, each adjusted weight shows the relative impact of each level in terms of standard deviation units for a scale having a mean of zero and a standard deviation of one. The values of the "constants" for each RDF (shown at the bottom of Table 19) represent the sum of all the means subtracted from the predicted factor scores. Technically, these constants could be added to the constants shown for the adjusted weights for Ss shown in Table 18. These adjusted weights and constants and the adjusted weights and constant for the Ss will produce exactly the same predicted factor scores that the **B**-weights produce. The advantages of the adjusted weights is that they enable determining the relative positions of all levels, even those not represented by separate predictor dichotomous variables.

4.3 Overview of Significant Experimental Manipulation Effects

The following subsections briefly discuss the significant experimental manipulation effects (i.e., Orders, Periods, Conditions, and interactions) studied and how they were interpreted for the various RDFs.

4.3.1 Order Effects

Only two RDFs had significant order effects ($p < .001$ level). They were RDF 1 ($O_1 = -.180$) and RDF 8 ($O_1 = -.249$). This indicates that performance on whatever these RDFs are, was significantly worse for the initial session than for subsequent sessions. Improvement across the various orders can be interpreted as a learning or practice effect as the study progressed.

4.3.2 Period Effects

According to the RDF Source Table (Table 11), RDF 2 and RDF 3 had significant ($p < .001$) period effects. The adjusted weights for these two RDFs were: RDF 2, P_1 (.149) and P_3 (-.149) and RDF 3, P_1 (-.082) and P_3 (.082). Whatever these RDFs represent, the direction of

these adjusted weights indicates that Ss, as each session progressed (i.e., from P_1 to P_3), performed relatively worse on RDF 1 and relatively better on RDF 3.

4.3.3 Period by Order Effects

Table 19 shows that none of the PxO terms reached significance at the .001 level. Indeed, most of the adjusted weights were extremely close to zero. The largest PxO adjusted weights were for RDF 8 for P_1O_1 (-.139) and P_3O_1 (.139). Although not significant, these weights indicate that Ss' performance on RDF 8 was, for whatever condition they underwent first, somewhat worse in period 1 than in period 3.

4.3.4 Condition and Period by Condition Effects

Only two RDFs attained significant ($p < .001$) PxC adjusted weights. They were: RDF 1, P_1C_3 (.139) and RDF 1, P_3C_3 (-.139) and RDF 3 for P_1C_3 (-.242) and P_3C_3 (.242). These RDFs were also the only ones that showed significant condition effects and they were, respectively, RDF 1; C_3 (-.270) and RDF 3; C_3 (.375). This finding suggests that Ss, after undergoing Condition 3 (more difficult tracking during P_2), exhibit less of whatever RDF 1 represents and more of whatever RDF 3 represents.

4.4 RDF 1: Ability to Time-Share Tasks

The four highest loadings on RDF 1 addressed (low) RMS Error on the tracking task:

RMSE in Elevation during the TA task	-.997
RMSE in Azimuth during the TA task	-.846
RMSE in Elevation during the Com task	-.848
RMSE in Azimuth during the Com task	-.976

Clearly, because the squares of these loadings were so close to one, this RDF explained most of the tracking error variance in both elevation and azimuth during the occasions in which either TA and Com tasks were being performed. The RDF also showed significant positive loadings for the four corresponding Stick Manipulation criteria:

Stick Manipulation in Elevation during the TA task	.429
Stick Manipulation in Azimuth during the TA task	.454
Stick Manipulation in Elevation during the Com task	.317
Stick Manipulation in Azimuth during the Com task	.376

The opposite signs of loadings for RMSE and Stick Manipulations were expected since Ss who manipulated their control stick more frequently would, in general, be expected to produce less tracking error. Because the loadings of all RMS Error were negative, the RDF indicates desired tracking performance. However, significant loadings for three of the four criteria measuring TA performance also loaded significantly on this RDF:

TA Percent Correct Responses	.552
TA Percent Incorrect Responses	-.469
TA Median Time for Incorrect Responses	-.266

Further, the signs of these loadings also indicated desired tactical assessment performance. While the three criteria for the Com task did not reach significance and were not nearly as high as those for tactical assessment, the signs of their non-zero loadings indicated that the RDF was also indicative of desired communications performance.

Had this RDF indicated desired performance for tactical assessment and communications, but undesired performance for tracking, the RDF would have indicated a trade-off of attention from the tracking task to the other tasks. However, since the loadings of some criteria for all three tasks indicated desired performance, the results suggested that Ss who did well in one task, especially tracking, also did well in the other two. The fact that TA Median Time for Correct Responses did not influence this factor ruled out the possible interpretation that the factor represented simply "rapid reaction time" or some "general decision-making speed." For all of

these reasons, and because individual differences were highly influential, this RDF was interpreted as a between-tasks type of RDF that measured the general ability to time-share among tasks.

RMS Error criteria had much higher influence on this RDF than did other criteria. This may be partly explained by the fact that tracking criteria (especially RMS Error) were continuously measured during a 3-minute period associated with immediately before and immediately after the TA or Com task began. Thus, RMSE criteria were probably far more reliable than any other criterion.

As it was shown earlier, this RDF had highly significant ($p< .001$) effects for Orders, Condition, and Period by Condition effects. The significant adjusted weights, however, were limited to O_1 (-.180), C_3, (-.270), P_1C_3 (.139), and P_3C_3 (-.139). The adjusted weight for O_1 indicates that Ss' time-sharing capability was, in general, worse in the beginning of the study regardless of what conditions were presented to them. This conclusion is supported by the fact that O_2 through O_7 (although not as significant) show a general performance improvement as the Ss gained more experience.

Condition 3 was one in which Ss performed all three tasks during P_2 (i.e., no automation) but tracking became more difficult during P_2. The adjusted weight for C_3 (-.270) was accompanied by the significant adjusted weights P_1C_3 (.139) and P_3C_3 (-.139). Together these weights indicated that performing the more difficult tracking during P_2 resulted in degraded ability to time-share among all three tasks during P_3. A simple interpretation of this finding is the effect of fatigue or relaxation following cessation of the more difficult tracking task in P_2.

4.5 RDF 2: Different Types of RMS Error for Com and TA Tasks

Only two criteria influenced this RDF significantly. These criteria were:

RMSE in Elevation during Com	-.503
RMSE in Azimuth during TA	-.406

These loadings indicated that Ss tended to make less elevation tracking error during the Com task and less azimuth tracking error during the TA task. This RDF had significant adjusted weights for P_1 (.149) and P_3 (-.149). The direction of these weights indicated that performance became worse from the beginning of each session (i.e., P_1) to the end of each session (i.e., P_3). As indicated earlier, individual differences significantly influenced this factor. The fact that the only significant loadings were present on tracking criteria indicates that this is a within-task RDF, although it is influenced by which other task is being performed.

4.6 RDF 3: Stick Manipulation Perseverance After Difficult Tracking

RDF 3 was a within-task factor since all four criterion variables influencing it addressed (increased) tracking task stick manipulations:

Stick Manipulations in Azimuth during the TA task	.835
Stick Manipulations in Elevation during the TA task	.587
Stick Manipulations in Elevation during the Com task	.712
Stick Manipulations in Azimuth during the Com task	.709

Individual differences significantly effected this factor. Significant effects on this RDF included Condition 3 (more difficult manual tracking during P2) and interactions of Condition 3 with Periods. The adjusted weights were C3 (.375), P1C3 (-.242), and P3C3 (.242). The more difficult tracking task during P2 apparently caused a greater demand for stick manipulations during that period. Apparently, the response of more stick manipulations carried over to P3 even after the tracking task had returned to its "easy" level. This finding suggested that Ss persevered in more stick activity immediately following a difficult tracking episode. The lack of significant loadings for any tracking RMS Error on this RDF indicated that the increased manipulations did not result in improved tracking (as measured by RMS Error). For the above reasons, this factor was interpreted as representing stick manipulation perseverance after difficult tracking. The greater number of stick manipulations could have been caused by either: (a) residual enhanced attention to the tracking task, (b) residual perceived need to rapidly null out error when Ss did attend to the tracking task, or both. This finding is important in that it offers evidence that prior task demands, even though not continuing currently, can significantly influence present task behavior.

4.7 RDF 4: Interference Between TA and Com Manual Responses

This RDF was a within-task factor since the only two significant criterion loadings on this factor addressed stick manipulations on the tracking task. These criteria were:

Stick Manipulations in Elevation during the Com task	.620
Stick Manipulations in Azimuth during the TA task	-.303

The directions and absolute magnitudes of these loadings suggested a factor that relates to more stick manipulations in Azimuth during a Com task than during a TA task. Order, Period, Condition, or their interactions did not have a significant effect. As shown earlier, the mean of predicted factor scores for Ss in the "Verbal-Response" group was not significantly different from the mean of the predicted factor scores for Ss in the "Manual-Response" group. Also, as shown earlier, individual differences played a significant role in the variance of this RDF.

4.8 RDF 5: Ability to Perform the Tactical Assessment Task

This RDF is another within-task factor since all three of the significant criterion loadings on this factor addressed the tactical assessment task:

Percent of Correct TA Responses	.536
Percent of Incorrect TA Responses	-.875
Median Time for Incorrect TA Responses	-.611

No experimental manipulations had significant effects. Individual differences appeared to be a significant determiner of this RDF and suggests that this RDF represents ability to perform the TA task. Assignment to the Verbal-Response mode for the Com task also had a significant effect, where its adjusted weight was .762 indicating that being able to respond verbally to the Com task may have permitted more visual attention to the TA task and, hence, improved performance.

The opposite signs for the criterion loadings for Percent of Correct TA Responses and Percent of Incorrect TA Responses were not surprising and simply indicated that those who make more correct responses will also make fewer incorrect responses. The high negative loadings for both the Percent of Incorrect TA Responses and the Median Time for Incorrect TA Responses, however, indicated a positive relationship between time to respond incorrectly and the number of incorrect responses. One explanation might be that the more difficult the TA task was, the longer Ss would take to respond to it, and the greater the likelihood of their being incorrect. Again, this supported the concept that this factor represents an ability to perform this task.

4.9 RDF 6: Ability to Perform the Communications Task

This RDF was also a within-task factor since the only criterion variables having significant loadings on it addressed the communications task:

Percent of Correct Com Responses	.974
Median Time for Incorrect Com Responses	-.779

As with the last RDF, these high opposite signs indicated that those who made more correct responses (i.e., less errors) tended to take less time when they did make errors. No experimental manipulation variables significantly effected this factor. Of all the common RDFs (i.e., between tasks or within-task types), this RDF had the lowest $\mathbf{R}^2$ value (.206) and was the least predictable. Individual differences accounted for about 28 percent of that variance and, since it accounts for nearly 95 percent of the variance in being correct on the Com task, this RDF represents the ability to perform the communications task.

4.10 RDF 7: Unwillingness to Verbally Interrupt a Verbal Message

This RDF was a between-tasks factor since criterion variables from two different tasks loaded significantly on it. Both also addressed the time to make correct responses:

Com Median RT Correct	-.973
TA Median RT Correct	.369

The opposite signs indicated something caused Ss who were rapid in providing correct responses to the Com task to be slow in providing correct responses to TA tasks, and vice versa. The square of the extremely high negative criterion loading for Com Median RT Correct (-.973) indicated that this RDF accounted for nearly 95 percent of that criterion's variance. Over 49 percent of this RDFs' variance was explained by individual differences and over 37 percent was explained by the Com response group to which the Ss were assigned. Ss having high adjusted weights on this factor would have short Com task response times. Order, Period, Conditions, and their interactions showed no significant effects.

The adjusted weight for the Verbal-Response Mode group was -1.233. This result indicated that Ss who responded verbally to the Com task took significantly longer to respond than those who responded manually. This finding was, initially, surprising since a verbal response to a verbal command would seem more compatible than a manual response to a verbal command. However, it should be remembered that the typical communications message in this study was a three-part statement such as, "Call-sign, change (altitude/heading) to a stated value." The S's task was, first, to decide if the call-sign heard was one assigned to him and, if so, to indicate whether the parameter to change was altitude or heading. It should be noted that the final part of the message (i.e., "to a stated value") was unimportant to making a correct response. A reasonable explanation of the results is that manual responses were initiated as soon as the appropriate call-sign and parameter were known while verbal responses were initiated only after the entire message had been heard. That is, voice discipline may have caused Ss who were responding verbally to begin their response only after the entire spoken message had been completed. If this was the case, then this RDF may be interpreted as an unwillingness to verbally interrupt a verbal message.

The two significant criteria had opposite signs. Two explanations are suggested for this finding. First, by chance alone, Ss in the Verbal-Response Mode group might have been, on the average, more skilled at the TA task and, consequently, correctly responded more rapidly on that task. Secondly, Ss responding verbally to the Com task may, because they did not have to think about how to manipulate the manual switch, have had more free time to scan the TA screen for targets.

4.11 RDFs 8 Through 15: The Within-Criterion Factors

Each of the remaining eight RDFs were, at most, influenced by only one significant criterion variable. This finding indicates that all of these RFDs were within-criterion factor types. The specific criterion variables associated with each of these eight RDFs and their highest loadings were:

Factor	Criterion Variable	Loading
8	TA Percent Correct Responses	.623
9	TA Median RT for Correct Responses	-.924
10	TA Median RT for Incorrect Responses	-.728
11	RMS Error in Azimuth during TA	-.302
12	Stick Manipulations in Elevation during TA	.641
13	Com Median RT for Incorrect Responses	-.569
14	RMS Error in Azimuth during Com	-.199
15	Stick Manipulations in Azimuth during Com	.585

Six of these RDFs were not significantly affected by any of the experimental manipulations (i.e., Order, Period, Condition, PxC, or PxO). RDF 8 was significantly affected by Order (i.e., tended to improve in TA Percent Correct Responses as the study progressed); RDF 11 was significantly affected by Condition (RMS Error in Azimuth during TA decreased in sessions having Conditions 2, 4, and 6). These were all conditions in which the TA task was at its "difficult" level during P_2.

Individual differences (i.e., Subject effects) were significant for three of these RDFs (i.e., 8, 9, and 15). Three of the four remaining factors (i.e., 10, 11, and 14) had shown significant **B**-weights for one or more Ss. Only factor 13 had no significant **B**-weights at the .01 level for any S. For the most part, the within-criterion factors can be interpreted as representing the remaining individual differences in the ability to perform a particular task and error variance.

4.12 Summary of the Significant Effects for the RDFs

An examination of the source table (Table 11) and the significant **B**-weights (shown in Table 13) for the 12 "Subject" variables shows that individual differences were a major determinant of behavior for 14 of the 15 rotated diagonal factors. Various experimental manipulations appear to have played a much smaller role as a determinant of behavior. For example, only "Order 1" and "Order 2" significantly impacted behavior (i.e., Ss improved their performance as the study progressed), but only for RDF 1 and RDF 8. Of the various conditions studied, Condition 3 had significant weights, but only for RDF 1 and RDF 3. "Period" played a significant role, but only for RDF 2 and RDF 3. The significant "Period by Condition" interactions were only significant for the same two RDFs where Condition 3 had an impact. Period by Order did not reach significance for any of the RDFs. Assignment of Ss to "Verbal-Response" or "Manual Response" modes for the communication task had a significant effect on RDF 5, RDF 7, and RDF 9.

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5. COMPARISON OF MANOVA AND RDF RESULTS

In this section, the results from the RDF and MANOVA approaches will be compared. This comparison will include the comparability of the overall significance of the **B**-weights, how the two approach differ with regard to their purposes, and the advantages and disadvantages of the two approaches.

5.1 Comparability of Overall Significance Results Based on B-Weights

To compare the results of the RDF approach with the MANOVA approach, a Multi-Criterion Multiple Correlation (MMC) program was used to obtain the **B**-weights and constants to be added (i.e., the model coefficients) for the 15 original criteria. Table 20 shows these **B**-weights and the levels of significance obtained for them.

As can be seen by examining the **B**-weights for the "Subject" variables in Table 20, performance on all 15 criteria were significantly impacted by individual differences. "Order 1" had a significant **B**-weight at the .001 level for five of the 15 criteria and "Order 2" had a significant **B**-weight at the .01 level for only one of the criteria. "Period 1" did not have a significant effect on any of the 15 criteria. Of the various conditions studied, only "Condition 3" had a significant impact at or beyond the .001 level. This occurred on six of the criteria all of which addressed the tracking task. Of all of the "Period by Condition" interaction variables' **B**-weights, only "Period 1 by Condition 3" reached significance at the $p < .001$ level. This result occurred on five of the 15 factors (all of which had also shown a significant "Condition 3" effect). None of the "Period by Order" interaction term **B**-weights reached significance at the .001 level. Being assigned to the Com task's "Verbal-Response Mode" had a significant impact on five of the 15 criteria at or beyond the .001 level and on one additional criterion at or beyond the .01 level.

With regard to the significance of **B**-weights for both individual differences and the four experimental manipulations (i.e., "Order 1," "Condition 3," "Period 1 by Condition 3," and "Verbal-Response Mode") that were significant at or beyond the .001 level, the results obtained were identical with those found by the RDF approach.

Most of the interpretations of significance made for both the RDF and MANOVA analyses were based on predictor **B**-weights (i.e., model coefficients or adjusted weights) that were significant at the .001 level. However, both analyses did show some predictors with **B**-weights which were significant at the .01 level as well. Considering only the 26 non-Subject predictors for the 15 factors, one would have expected 3.9 (=.01 x 26 x 15) variables to have reached the .01 level by chance alone. Since only seven of the MANOVA model coefficients did reach that level of significance (i.e., between .01 and .001), many if not all of them may have occurred by chance alone. By comparison, only one RDF **B**-weight was at that level of significance (i.e., between .01 and .001). Clearly, it may have occurred by chance alone. For this reason, significance of **B**-weights between the .01 and .001 might not be deemed adequate for attempting to interpret the factors.

Table 20. MANOVA Model Coefficients for the 15 Criteria

Variable	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12	Y13	Y14	Y15
Verb-Resp. Mode	.22	**-.25**	-.01	.09	.74	2.02	*.15*	**-.22**	.00	**1.41**	.00	2.52	1.57	**.20**	*-.14*
Subject V1	-2.81	**-.22**	1.52	-.07	**16.23**	**15.79**	**-.22**	**-.27**	**-5.24**	-.05	**1.37**	**14.91**	**14.19**	**-.23**	**-.27**
Subject V2	-1.51	.01	.22	-.09	**5.47**	**3.47**	-.01	.07	-1.51	**-.27**	.79	**2.98**	**5.22**	.05	.01
Subject V3	*-3.67*	-.01	.44	-.09	**18.41**	**7.52**	**-.23**	-.10	-3.16	-.11	.76	**6.07**	**18.69**	**-.25**	*-.10*
Subject V4	-2.65	**.21**	1.09	-.22	**16.83**	**9.43**	**-.24**	-.22	-2.81	-.09	*1.18*	**8.39**	**16.81**	**-.25**	**-.23**
Subject V5	1.74	-.08	-.43	-.55	-1.15	-1.10	**-.27**	-.10	-2.22	*-.15*	.79	-1.68	-.51	**-.30**	**-.18**
Subject V6	-.86	-.00	.87	-.03	1.90	.31	.01	**.18**	**-4.47**	**-1.30**	.93	-.65	1.05	-.03	.05
Subject V7	*-3.27*	-.02	.66	-.21	2.30	-.19	-.06	**.21**	.00	**.18**	.00	-1.76	2.33	**-.12**	*.09*
Subject M1	-2.41	**-.23**	*1.97*	.47	**9.10**	**7.30**	-.04	**-.24**	.00	**.27**	.00	**6.66**	**8.66**	.03	**-.23**
Subject M2	**-4.57**	**-.27**	**2.82**	.45	2.66	*2.67*	.03	**-.22**	.00	**1.18**	.00	1.62	*3.10*	.03	**-.20**
Subject M3	*-3.25*	.03	-.44	-.45	**6.69**	**6.25**	.08	**-.33**	.00	-.01	.00	**5.97**	**7.34**	**.17**	**-.13**
Subject M4	**-21.19**	-.10	**11.81**	**1.15**	**20.35**	**11.22**	**-.31**	**-.61**	.00	**.70**	.00	**11.28**	**21.03**	**-.22**	**-.53**
Subject M5	**-9.47**	-.03	**2.41**	.72	**9.47**	**10.20**	.08	**-.24**	-.71	.02	.39	**9.39**	**8.96**	**.13**	**-.25**
Order 1	**-6.13**	.02	2.01	.52	**6.34**	**4.91**	-.14	-.14	-.85	.05	-.03	**5.28**	**7.24**	-.11	-.09
Order 2	-.63	.10	.74	.06	*5.68*	2.55	-.01	-.09	1.45	.10	-.16	2.62	4.51	-.09	-.08
Order 3	-.92	.07	.06	-.11	2.28	1.84	.01	-.05	-1.18	.04	-.00	1.34	1.53	-.05	-.09
Order 4	-2.33	.07	.26	-.00	1.80	.95	.05	.03	-1.39	-.08	.18	1.33	2.45	-.02	-.02
Order 5	-2.02	.11	.69	-.27	2.14	1.30	.02	-.07	1.40	.02	-.44	1.38	1.54	-.04	-.10
Order 6	.59	.05	-.64	-.18	.29	.25	.05	.03	1.28	.08	-.55	-.01	-.75	-.04	-.01
Period 1	-.66	-.06	.90	.09	-.34	-1.34	.05	-.10	-.58	.10	.12	-1.85	-1.48	-.02	*-.15*
Condition 1	.30	.02	-.49	-.17	1.07	.69	-.08	-.08	-1.59	.09	.11	-.42	-.25	.01	-.03
Condition 2	.45	-.02	.36	-.21	-2.14	-3.14	-.12	-.14	-1.03	.00	.32	-1.32	-.11	-.00	*-.13*
Condition 3	-2.16	-.05	1.67	.41	**12.24**	**10.25**	.11	*.20*	-.89	.06	.46	**8.18**	**10.50**	**.24**	**.21**
Condition 4	-1.86	.04	-.12	.08	-2.94	*-3.28*	.01	.03	-.71	.03	.23	-1.91	-2.01	.05	.07
Condition 5	-.65	-.05	-.31	-.18	.31	-1.59	.08	.06	-3.77	-.01	1.07	-.86	-.52	-.03	-.02
Condition 6	1.56	.02	-.94	-.45	-3.14	*-3.33*	-.05	-.03	-1.76	-.06	.86	-1.33	-1.24	-.03	.01
P1 x C1	1.43	-.04	-.61	-.11	-.99	-.33	.04	.06	.16	-.12	.57	-.01	-.71	.02	.08
P1 x C2	-1.06	-.03	-1.86	-.28	.56	1.98	.16	.22	-.42	-.16	.12	1.50	-.62	.02	.12
P1 x C3	2.47	-.08	-2.06	-.79	**-11.7**	**-9.10**	-.16	-.14	-1.83	-.24	.89	**-7.33**	**-10.10**	**-.29**	-.15
P1 x C4	2.93	-.03	-.58	-.26	1.40	-.40	.02	.13	-.56	-.21	.30	.06	1.59	-.01	-.04
P1 x C5	-1.92	.07	-1.40	.19	1.30	2.35	-.10	.09	-3.03	.05	.39	1.10	1.73	.04	.06
P1 x C6	-.65	-.03	.28	.38	.09	.76	.02	.09	-.75	.04	.10	.88	-.30	.07	.02
P1 x O1	*-7.62*	.06	1.69	.43	3.38	-.45	.03	.12	1.38	.16	-.74	-.89	3.31	.06	.12
P1 x O2	-.24	-.03	.44	.07	.50	.90	-.11	-.03	3.06	-.14	-.89	.27	1.75	.02	.13
P1 x O3	.81	.02	.86	.56	.37	-1.23	-.10	-.04	2.84	.01	-.29	-.63	1.26	.07	.08
P1 x O4	1.93	.07	.17	.01	.31	.51	-.09	-.08	4.46	.25	-1.45	.33	-.33	-.00	.06
P1 x O5	2.69	.04	-1.04	.00	.78	.35	-.04	-.00	2.31	.01	-.79	.51	.95	.03	.13
P1 x O6	-.12	-.03	1.30	.13	1.32	.80	-.04	-.04	.88	-.07	-.22	.99	2.67	.01	.09
Constant	100.07	1.65	-.06	.57	6.46	7.10	.51	1.05	100.98	1.18	-.21	6.09	5.42	.48	1.01

Notes: Bold = significant < .001 level; italics < .01; Ss with .000 on criteria 9 and 11 did not exhibit Com errors.

5.2 Conservation of Information by the RDF Approach

No information available in the MANOVA approach is lost by using the RDF approach. This conclusion is supported by the fact that all MANOVA model coefficients for any predictor **i** and any criterion **j** may be reconstructed from the criterion loadings in the RDF matrix (i.e., in matrix [$\mathbf{F}_{YN}$] as shown in Table 7) and the **B**-weights for predicting those factors as seen in Table 13. The model coefficients of variable **i** for predicting criterion **j** can be computed by obtaining the product of the standard deviation (**s**) of criterion **j** times the sum (across all **n** factors) of the product of predictor **i**'s **B**-weights and criterion **j**'s factor loadings. That is,

$$B_{ij} \quad = \quad s_j \sum_{k=1} (B_{ik} \; f_{jk}) \, ,$$

where:

B_{ij} = the MANOVA **B**-weight for predictor **i** for criterion variable **j**
s_j = the standard deviation of criterion variable **j**
B_{ik} = the RDF **B**-weights for predictor variable **i** for RDF **k**
f_{jk} = the loading of predictor variable **i** on RDF **k**

Once the **B**-weights for the criterion variables are determined, the "constant to be added" for predicting criterion **j** can be computed by the equation:

$$C_j \quad = \quad M_j - (B_{ij} \, M_i) \, ,$$

where:

C_j = the MANOVA constant to be added for the prediction of criterion **j**
M_j = the mean of criterion variable **j**
B_{ij} = the **B**-weight for predictor **i** for criterion variable **j**
M_i = the mean of predictor variable **i**

These equations provide further proof that the final solutions arrived at by the RDF approach can fully explain and account for any MANOVA solution.

5.3 Contrast Between the Purposes of RDF and MANOVA Approaches

The purpose of the MANOVA approach, for a set of **m** possibly related criteria, is: (a) to determine the model coefficients applicable for predicting each criterion variable and (b) to determine the likelihood that subject and experimental manipulations played a significant role in determining performance behavior across those criteria. The method of determining significant effects for MANOVA is different from that used in a single ANOVA because the various criteria in the criterion set may be related to each other.

The RDF approach recasts the total variance of the **m** related criteria into **m** independent dimensions (i.e., factors). These independent factors fully explain the intercorrelations among the criterion variables as well as the relationships between the criterion variables and the predictor variables (i.e., subject and experimental manipulations). Orthogonal rotations performed on these independent factors will also explain all the variance of the criteria, their intercorrelations, and their relationships to the predictors. However, these rotations permit the investigator to find a more meaningful set of independent factors (i.e., the rotated diagonal factors) that will help explain: (a) why the criterion variables are related as they are and (b) how these factors were influenced by both subject and experimental manipulations. The purpose of the RDF approach is: (a) to determine the model coefficients applicable for predicting each RDF and (b) to determine the likelihood that subject and experimental manipulations played a significant role in determining performance behavior on those independent factors.

5.4 Advantages of the RDF Approach

The RDF approach is not applicable to studies in which data are collected on a single criterion. It is, however, applicable for multi-task, multi-criterion studies. If investigators are only interested in testing hypotheses related to the overall impact of individual differences and experimental manipulations on each separate criterion, then MANOVA is adequate for that purpose. However, if an investigator is also interested in understanding how (and why) separate performance criteria for one or more tasks are related, then some sort of factor analytic approach

is indicated. The RDF approach appears to provide an analytical technique that bridges the gulf between traditional ANOVA techniques and traditional factor analytic techniques. RDF can be used not only to determine both independent common factors (i.e., between-tasks and within-task factors) and unique factors (i.e., within-criterion factors), but also to provide the model coefficients for each independent factor. In this way, the investigator can use the RDF approach to isolate and better understand various independent factors that simultaneously effect performance on two or more tasks, two or more criteria on a single task, or a single criterion on a single task.

5.5 Disadvantages of the RDF Approach

From a theoretical basis, the RDF approach is superior to MANOVA. However, its approach currently has one practical disadvantage. This disadvantage is that the RDF approach requires graphical rotation of factors. Computer programs are available in many commercially available statistical programs for accomplishing MANOVA and they require little if any human interaction. However, most of these packages do not contain procedures for obtaining diagonal factors or for performing graphical rotation. Further, many experimental psychologists have had only introductory courses in factor analysis and have little or no practical experience in how graphical rotation is accomplished.

Graphical rotation can be accomplished on a computer screen by having the computer plot the current loadings of one selected factor at right angles against the current loadings on a second selected factor and then letting the human determine the best angle of rotation. Since the factors must be graphically rotated in pairs, the total number of graphical rotations that must be done with **K** factors just to examine and rotate each pair of factors once is $\mathbf{K(K-1)/2}$. Further, since each rotation results in slightly different loadings on both factors, it is necessary to rotate the set of factors many times before a final and satisfactory solution (i.e., one that cannot be significantly improved) is reached. If we assume that the entire set of factors must be rotated at least **K-1** times, then the total number of two-factor rotations required will be $\mathbf{K(K-1)^2/2}$. The study presented here had 15 factors and required approximately 1470 graphical rotations. The

interactive computer program used in this study for graphical rotation was extremely fast in plotting the next pair of factors. A mouse was used to indicate the new, desired angle of rotation. However, even if the average decision time for rotating each pair of factors could be accomplished in five seconds, 1470 such decisions would require over two hours of fairly intensive effort.

Varimax rotation can be accomplished without human interaction. It also rotates factors two at a time and iterates its solution until it cannot improve it based on some mathematical criteria. Indeed, to reach a stable Varimax solution would also require a similar number of rotations, but a computer can perform these rotations very rapidly without human intervention. The mathematics of Varimax rotation is based on the concept that the factors being rotated will ultimately exhibit simple structure. However, simple structure cannot be found when between-tasks, within-task, and within-criterion factors exist simultaneously. Varimax rotation was attempted on the 15 diagonal factors: part of the variance of each between-task factor had been rotated on to each within-task and within-criterion factor that had high loadings on the same tasks as those between-task factors. Similarly, part of the variance of each within-task factor had been rotated on to each within-criterion factor that had high loadings on those within-task factors. The net result from Varimax rotation was an unsatisfactory solution to finding the desired factorial structure. Because of this problem, the diagonal factors had to be rotated graphically (i.e., with the author deciding how each rotation should be done). Investigation on methods to automate the rotation of these factors is underway.

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6. SUMMARY AND CONCLUSIONS

In situations where Ss are required to perform multiple ongoing tasks, researcher should be concerned with the ability of Ss to attend to and handle the multiple task responsibilities as well as how Ss may be trading-off performance among multiple criteria for the different tasks. To answer such concerns, one must go beyond the typical MANOVA analyses currently in vogue for multiple criteria studies. The RDF approach discussed in this paper was developed to permit investigators to analyze their experimental data to determine the number and nature of independent dimensions that cause various criteria to covary and to determine the significance of individual differences and experimental manipulation effects on the independent dimensions found. The RDF approach involves the extraction of criterion-based diagonal factors and their rotation to a task- and criterion-based structure. The results of this rotation is a set of rotated diagonal factors (RDFs).

Theoretically, there are only three possible task- and criterion-based types of independent RDFs that can be found. The first is a between-tasks type that accounts for significant criterion correlations among different tasks. The second is a within-task type that accounts for significant relationships between criteria for the same task that are not accounted for by the first type. The third and final type is a within-criterion type that accounts for the remaining variance of a criterion that is not accounted for by either of the first two types. It is feasible that both individual differences and experimental manipulations could significantly impact all three types of RDFs.

The RDF methodology was applied in the analysis of results obtained from a study investigating subject performance of several tasks. In the study described herein, two between-tasks RDFs were found. The first between-tasks RDF appeared to account for much of the variance of many of the criteria and was interpreted as a general ability to time-share among tasks. The analysis showed that Ss improved significantly in this ability as they continued in the study. The analysis also showed that, when continuous tracking became more difficult for a period of time, Ss, in general, tended to continue to attend more to the tracking task, even after it returned to its original less difficult level.

The second between-task RDF was far less important in terms of criterion variance explained. It dealt with response time for correct responses for both the tactical assessment task and the communication task. This RDF indicated that a significant negative relationship (-.359 = -.973 x .369) existed between these two criteria. Highly significant **B**-weights (with opposite signs) for different Ss show that some Ss were better at one of the discrete tasks than the other. This outcome could have been caused by Ss possessing different skills for the two discrete tasks or by some Ss paying more attention to one task than to the other. Thus, this between-task factor may indicate the trade-off of attention among competing tasks.

Five within-task RDFs were also identified. These RDFs explained all of the significant criterion interrelationships for a given task that were unexplained by the two between-task RDFs. Three of these factors dealt with the tracking task's criteria; one dealt with the tactical assessment task's criteria, and the remaining one dealt with the communications task's criteria.

Finally, on the eight within-criterion RDFs found, **B**-weights for Ss showed that individual differences had a significant impact on the single, specific criterion associated with that factor.

With the RDF approach, each type of independent RDF found, as has been seen, can be tested for the significance of individual differences. Additionally, each type of independent factor can also be tested for the significance of various experimental manipulations (e.g., Order, Conditions, Period, and Com task response mode), and their interactions. In this study, it was shown, for example, that Condition 3 (when tracking became more difficult during Period 2) had a significant impact on two independent behaviors. It had a significant impact on the ability of Ss to time-share among tasks (a between-tasks factor) and it resulted in Ss persevering in making more stick manipulations even after the tracking task ceased to be difficult (a within-task factor). Thus, while investigators may have traditionally thought of experimental manipulations as having a unitary effect, this study has shown that at least some experimental manipulations can have significant effects on more than one independent dimension of performance.

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